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# Combining Cuckoo and Tabu Algorithms for Solving Quadratic Assignment Problems

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**Abstract**
Quadratic Assignment Problem (QAP) is one the combinatorial optimization problems. Research on this type of problems has been done in many companies for allocating some facilities to some locations. The issue of particular importance in this process is the costs of this allocation. Therefore, the attempt in this problem is to minimize this group of costs. Since the QAP's are from NP-hard problem, they cannot be solved by exact solution methods. Meta-heuristic methods give acceptable solutions in sensible times to NP-hard problems in engineering and science fields. In this work, QAP was first solved by themeta-heuristic Cuckoo algorithm, and then this algorithm was combined with the Tabu algorithm and the results were compared. It is shown that the combination of Cuckoo and Tabu algorithms leads to more optimized solutions. The results were also compared with other meta-heuristic algorithms and the results show that the combination of Cuckoo and Tabu algorithms is better than other single algorithms.

**Keywords: Optimization problems, Cuckoo algorithm, Quadratic Assignment Problem (QAP), Meta-heuristic algorithms, Tabu algorithm, NP-hard problems**

## 1. Introduction

In 1957, Koopmans and Beckmann introduced the Quadratic Assignment Problem (QAP) for the first time as a mathematical model related to economic activities[1]. QAP is one of the most difficult problems of combinatorial optimization and deals with allocation of a set of facilities, machines, or units to a set of locations or activities with minimal cost. Planning the space, grading archaeological data, arranging hospitals, designing keyboards for typists, etc. are some examples of the application of QAP. Assigning *n* facilities to *n* locations is proportional to the flow between the facilities multiplied with their distances is done with the purpose of allocating each facility to each location in such a way that the total cost is minimized. Therefore, there will be two $n*n$ matrices: the flow matrix, $A=(a_{ij})$ and the distance matrix $B=(b_{ij})$.

$$\min_{\pi \in S_n} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{\pi(i)\pi(j)} b_{ij} \tag{1}$$

Where $S_n$ is a set of permutation of {1, 2, 3,…, n}. Any individual product of $a_{\pi(i)\pi(j)}b_{ij}$ is the cost of assigning facility $\pi(i)$ to location $i$ and facility $\pi(j)$ to location $j$. A QAP with input matrix A, B is shown as (A, B). Sometimes, if any of the coefficient matrices A, B is symmetric, QAP (A, B) is called symmetric. Otherwise, it is called asymmetric QAP [2].

Another problem, which is a little different and has been investigated by several authors, is also taken as a QAP as follows. In addition to the two coefficient matrices A and B, a third matrix $C=(c_{ij})$ is given where $c_{ij}$ is the cost of placing facility $i$ at location $j$, and the problem will be:

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$$\min_{\pi \in S_n} \sum_{i=1}^{n}\sum_{j=1}^{n} a_{\pi(i)\pi(j)} b_{ij} + \sum_{i=1}^{n} c_{\pi(i)i} \tag{2}$$

There are two major groups of methods for solving optimization problems: exact methods and meta-heuristic methods. Some exact algorithms for solving QAP include dynamic programming [3] and branch and bound family algorithms [4, 5]. Exact methods determine optimum solutions and fulfill the optimization condition. However, problems with sizes greater than 20 are not usually solvable by exact methods, thus calling for meta-heuristic methods produces high quality solutions in a sensible time but do not guarantee finding the most optimized overall solution. Meta-heuristic algorithms includes construction methods [6, 7], limited enumeration methods [8, 9], improvement methods [10], simulated annealing methods [11], tabu search [12, 13], genetic algorithm [14], greedy randomized adaptive search procedure [15], ant colonies [16, 17], and imperialist competitive algorithm [18]. Here, the cuckoo optimization algorithm for solving QAP is explained.

## 2. Cuckoo Optimization Algorithm

Cuckoo optimization, anmeta-heuristic algorithm inspired from the nature, was developed by Yang and Deb in 2009 [19]. Cuckoo Optimization Algorithm (COA) was introduced by Rajabioun in 2011 [20]. COA is a novel continuous overall conscious search inspired from the life of a bird known as cuckoo. COA starts with an initial population, like other meta-heuristic methods; a population of cuckoos. These cuckoos have some eggs which should be laid in the nests of some host birds. The eggs which are more similar to the eggs of the host bird, have higher chances of growing up and becoming mature cuckoos, while other eggs will be identified and killed by the host bird. The number of the eggs, which grow up, shows suitability of the nests in that area. More number of the eggs that survive and live in an area have more tendency to allocate in that area. The situation in which more eggs survive is the parameter which COA wants to optimize. To maximize the survival of their eggs, cuckoos look for the most suitable area. Each cuckoo randomly lays some eggs in nests of host birds in its Egg Laying Radius (ELR). ELR for each cuckoo is defined by the following equation:

$$ELR = \alpha \times \frac{Number\ of\ \ current\ cuckoo's\ \ eggs}{Total\ number\ of\ \ eggs} \times (\mathrm{var}_{hi} - \mathrm{var}_{low}) \tag{3}$$

When all the cuckoos lay their eggs, some of the eggs, which are less similar to the eggs of the host bird, are recognized and thrown out of the nest. Therefore, after each egg laying, P% of all the eggs (usually 10%), whose profit function value is smaller, die. The rest of the young cuckoos are fed and raised in the host nests. When the young cuckoos grow up and become mature, they live for a while in their habitats and groups, but as the egg-laying time approaches, they migrate to better habitats where the eggs have higher survival chances. Once cuckoo groups are formed in all different parts of the environment (search area of the problem), the group with the best location is chosen as the objective point for all other cuckoos for migration. When mature cuckoos live all over the habitat, it is a difficult to figure out to which group each cuckoo belongs. To solve this problem, grouping of cuckoos is done by K-means clustering method (one k between 3 to 5 is sufficient). Now that cuckoo groups are formed, the average profit of the group is calculated to give the group's relative optimization of habitat. Then the group with the greatest average profit (optimization) is selected as the goal group to which the other groups migrate. In migration to the goal habitat point, the cuckoos do not travel the entire path; they just travel part of the path even in which they have deviational paths. This travel behavior is clearly shown in Figure1:

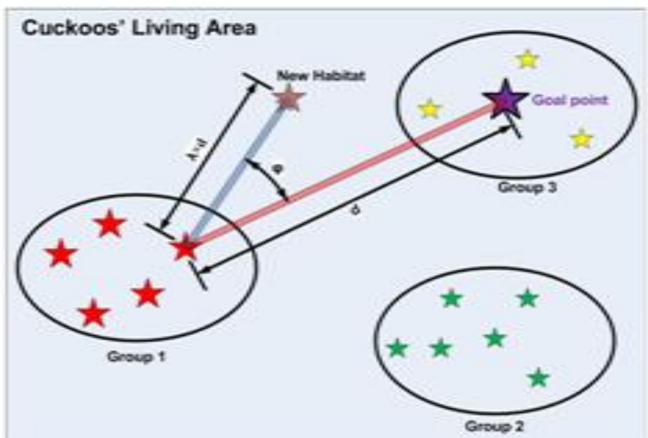


**Figure 1:**Migration behavior of the cuckoos [20]

As it can be seen in the above figure, each cuckoo only travels λ% of the entire path to the current ideal goal and has a deviation of φ radians. These two parameters help the cuckoos search a larger area. λ is a random number between 0 and 1 and φ is a number between π/6 and –π/6.Once all the cuckoos have migrated to the goal point and all the new habitats are determined, each cuckoo lays some eggs. Based on the number of eggs of each cuckoo, an ELR is determined for it and then, egg laying starts. Considering the fact that there is always a balance in the population of the birds, the maximum number of the cuckoos that can live in a habitat is limited to a number such as $N_{max}$. This balance is due to food limitations, being hunted by the hunters, and also the likelihood of not finding suitable nests for the eggs. After some repeats, all of the cuckoos reach an optimized point with the maximum similarity of the eggs to the eggs of the host bird as well as the maximum food sources. This place will have the greatest profit and the number of the eggs which die in it will be minimal. Convergence of more than 95% of all the cuckoos towards a single point takes COA to its end. Relationship (4), migration function in COA, is:

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$$X_{NextHabitat} = X_{CurrentHabitat} + F \times (X_{GoalPoint} - X_{CurrentHabitat}) \quad (4)$$

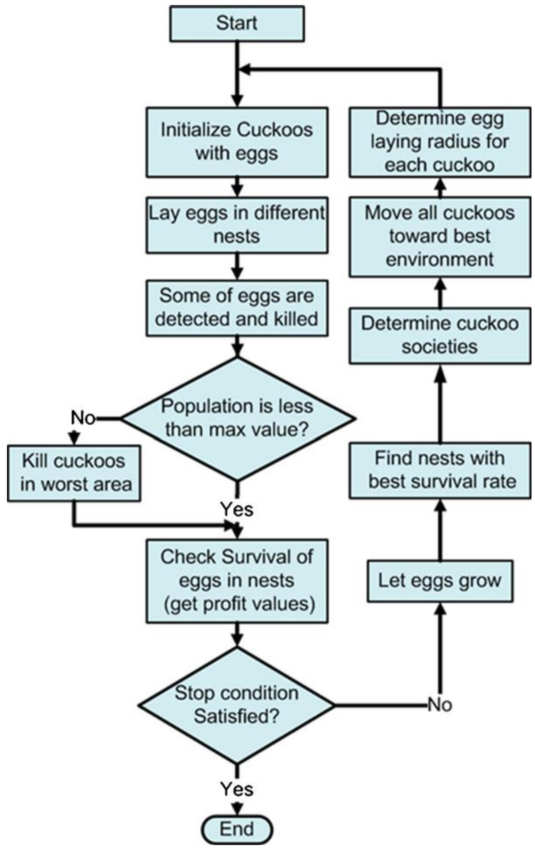


**Figure 2:** Flowchart of Cuckoo Optimization Algorithm

## 3. Tabu Search Algorithm

Tabu Search (TS) algorithm is anmeta-heuristic optimization algorithm which was first introduced by Glover in 1986 [21]. The word tabu, which is taken from Tongan, the language of the residents of Polynesia islands in the Pacific Ocean, means “a sacred object that should not be touched due to its sacredness”. According to Webster’s Dictionary, today, this word means “forbidden because of being risky”. The latter meaning of the word tabu completely matches the technique of TS. The risk, which is tried to be avoided, is the danger of improper paths.

To reach an optimized solution in an optimization problem, TS starts from an initial solution. In each run of the algorithm, a neighborhood is defined for the solution. Then the algorithm chooses the best neighboring solution among the neighbors of the current solution. If this solution is not in the forbidden list, the algorithm moves to the neighboring solution. Otherwise, the algorithm will check a so-called “aspiration criterion”. Based on the aspiration criterion, if the neighboring solution is better than the best solution found so far, the algorithm will move to it even if that solution is in the forbidden list.

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After the movement of the algorithm to the neighboring solution, the forbidden list is updated. That is, the previous move with which we moved to the neighboring solution is put in the forbidden list to avoid the returning of the algorithm to that solution making a loop. In fact, the forbidden list the tools by which the algorithm is prevented from visiting the local optimum. After putting the previous move in the forbidden list, some the moves that were put in the forbidden list before are excluded from the list. The time that the moves stay in the forbidden list is determined by a parameter called forbidden time. Moving from the current solution to the neighboring solution continues up to the point when the termination condition is met. Different termination conditions can be assumed for the algorithm. For example, limiting the number of moves to the neighboring solution can be a termination condition.

**Table 1:** Results of running the program with 10 repeats for the problems chosen from QAPLIB. Errors are in percent.

| COA-TS Error | COA Error | HBMO Error | Errors of Previous Methods | Solution Method | Best Solution Found So Far | Problem Size | Problem Name |
|---|---|---|---|---|---|---|---|
| 1.92 | 2.01 | 3.74 | - | Exact | 13178 • | 30 | **Lipa30a** |
| 0.88 | 0.95 | 2.25 | - | Exact | 107218 • | 60 | **Lipa60a** |
| 0.361 | 0.39 | 1.67 | - | Exact | 360630 • | 90 | **Lipa90a** |
| 3.22 | 3.56 | 16.11 | 5/91 | RO-TS | 23386 • | 49 | **Sko49** |
| 3.381 | 4.39 | 18.49 | 5/37 | RO-TS | 34458 • | 56 | **Sko56** |
| 4.34 | 4.47 | 16.91 | 5/7 | RO-TS | 48498 • | 64 | **Sko64** |
| 4.10 | 4.85 | 14.34 | 5/38 | RO-TS | 66256 • | 72 | **Sko72** |

## 4. Experiments and Simulation Results

This study addresses the general form of QAP. The purpose of this study is to evaluate the behavior of COA in solving QAP. So that its applicability is confirmed and it can then be used to solve the specific real cases in the next studies. For this reason, those problems which are more famous and have been used for testing other algorithms are chosen. Therefore, the most credible reference of QAPs, QAPLIB, was used which was prepared by Peter Hann, Berkard, Chella, Randal, and Karisch who are mathematics professors that specialize in QAP. In QAPLIB, different QAPs in several sizes are defined and solved by scientists such as Berkard, Al-Shaafi, Steinburg and etc, using exact, heuristic, and meta-heuristic methods.

First, we determine the results for a small population. Then, the resulting error percent is calculated and compared to the genetic and honeybee improved algorithms. The following results were achieved with at most 200 repeats and initial cuckoo population of 5 with at most 5 eggs for each bird. The maximum number of the cuckoos possible for living in each step is determined as 30.

**Table 2:** Results of running the program with 100 repeats for the problems chosen from QAPLIB. Errors are in percent.

| COA-TS Error | COA Error | HBMO Error | Errors of Previous Methods | Solution Method | Best Solution Found So Far | Problem Size | Problem Name |
|---|---|---|---|---|---|---|---|
| 1.901 | 2.007 | 3.78 | - | Exact | 13178 | 30 | **Lipa30a** |
| 0.85 | 0.96 | 2.3 | - | Exact | 107218 | 60 | **Lipa60a** |
| 0.35 | 0.38 | 1.65 | - | Exact | 360630 | 90 | **Lipa90a** |
| 3.19 | 3.55 | 18.82 | 5/91 | RO-TS | 23386 | 49 | **Sko49** |
| 3.380 | 4.388 | 15.88 | 5/37 | RO-TS | 34458 | 56 | **Sko56** |
| 4.325 | 4.459 | 14.36 | 5/7 | RO-TS | 48498 | 64 | **Sko64** |
| 4.093 | 4.78 | 13.78 | 5/38 | RO-TS | 66256 | 72 | **Sko72** |

**Table 3:** Results of running the program for the problems chosen from QAPLIB with large population.

| Error of the Best COA-TSSolution (%) | Error of the Best COASolution (%) | Error the Best HBMO Solution (%) | Error of the Best HGA Solution (%) | Problem Size | Optimized Solution | Problem Name |
|---|---|---|---|---|---|---|
| 13.84 | 18.46 | 54.86 | 21.69 | 32 | 130 | Esc32a |
| 11.90 | 17.04 | 50.56 | 20.75 | 32 | 168 | Esc32b |
| 0 | 0 | 9.7 | 0 | 32 | 642 | Esc32c |
| 2 | 3 | 29.57 | 0 | 32 | 200 | Esc32d |
| 0 | 0 | 0 | 0 | 32 | 2 | Esc32e |
| 0.4566 | 8.21 | 22.06 | 1.79 | 32 | 438 | Esc32h |

As it can be seen, COA has given a better and more optimized solution compared to honeybee and genetic algorithms.

## 5. Conclusion

In this paper, using the combination of COA and TS methods for solving QAP was studied. The results were determined for small, medium, and large populations. Moreover, the effect of the number of optimization repeats on precision of the solutions was investigated. The results suggest that COA separately works better than honeybee and genetic algorithms. However, combination of this algorithm with a local search algorithm called tabu search increases the optimization precision considerably. While the errors of genetic and honeybee algorithms had a gradual increase in problems of medium size, COA combined with TS still produced suitable solutions for problems of large size.

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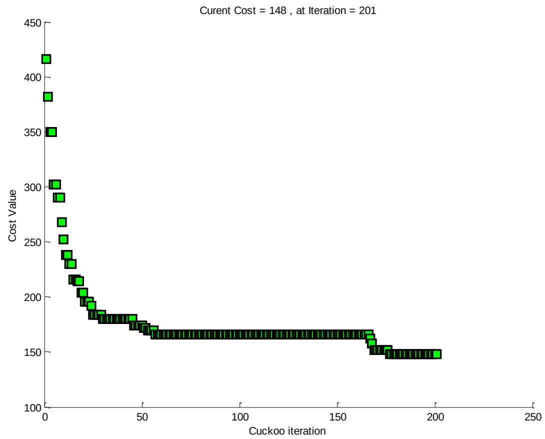


**Figure 3:** Convergence of COA for esc32a

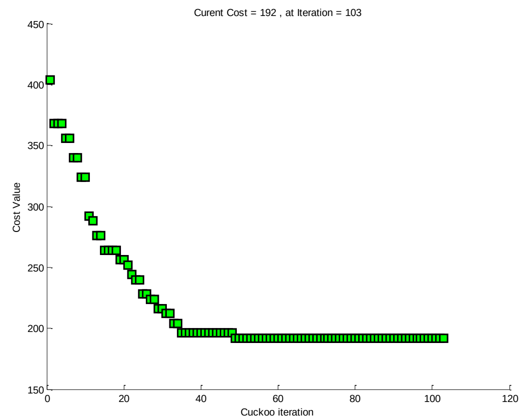


**Figure 4**: Convergence of COA for esc32b

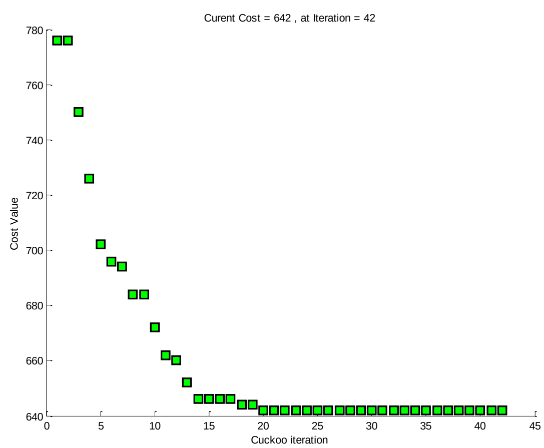


**Figure 5:** Convergence of COA for esc32c

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