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Novel Definition of Cost Function for Camera Calibration using Vanishing Point Theorem

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Abstract

Camera calibration is the process of determining the internal camera parameters and optical characteristics (intrinsic parameters). It also determines the relation between the two dimensional (2D) image and the corresponding three dimensional (3D) real points. Intrinsic camera parameters include: Focal Length, Principal Point, and Lens Distortion. In this paper, for estimating the intrinsic camera parameters, a simple and novel cost function is proposed based on the spatial geometry principles. Minimizing the proposed cost function, results in an accurate and simultaneous estimation of intrinsic camera parameters. Nonlinear Programming is used as the optimization method and the result shows the effectiveness of our proposed method.

Keywords: Camera calibration, intrinsic parameters, Nonlinear Programming, Optimization, Vanishing point.

I. Introduction

Camera calibration is an essential step in machine vision and photogrammetric applications such as robotics, three dimensional (3D) reconstructions; the performance of a vision inspection system is often strongly dependent upon the calibration accuracy. The computed camera parameters can then relate the location of pixels in the image to object points in the 3D reference coordinate system.

The existing camera calibration techniques can be classified into linear approaches (Abdel-Aziz, 1971; Okamoto, 1981) and nonlinear approaches (Wong, 1975; Faig, 1975; Sobel, 1974, Paquette, 1990). Linear methods have the advantage of computational efficiency but suffer from a lack of accuracy and robustness. Nonlinear methods, on the other hand, offer a more accurate and robust solution but are computationally intensive and require good initial estimates

The other common strategy for camera calibration is a two-step method given in (Weng et al., 1992). The first step generates an approximate solution using a linear approach, and this solution is improved by using a nonlinear iterative process in the second step.

Tsai's two-step method (Tsai, 1987) and Zhang's plane method (Zhang, 2000) solve some parameters linearly, then consider distortion and solve the remaining parameters using a nonlinear optimization method; The calibration accuracy of both methods are acceptable, but Tsai's two-step method does not calibrate the principal point and the ratio of horizontal-vertical coordinates. Zhang’s plane method requires camera to shoot calibration object from

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various angles. Reference (Wei xing and Feng, 2006) proposes a linear calibration method basing on coplanar, but the method ignores the lens distortion and the calibration accuracy is low.
The first step utilizing linear approaches is a key to the success of two-step methods. Approximate solutions provided by the linear techniques must be good enough for the subsequent nonlinear techniques to correctly converge. Being susceptible to noise in image coordinates, the existing linear techniques are, however, notorious for their lack of robustness and accuracy (Wang and Xu, 1996). In ref (Haralick et al., 1989) shows that when the noise level exceeds a knee level or the number of points is below a knee level, these methods become extremely unstable and the errors diverge.

The use of more points can help alleviate this problem. However, fabrication of more control points often proves to be difficult, expensive, and time-consuming. For applications with a limited number of control points, e.g., close to the required minimum number, it is questionable whether linear methods can consistently and robustly provide well enough initial guesses for the subsequent nonlinear procedure to correctly converge to the optimal solution. Another problem is that almost all nonlinear techniques employed in the second step use variants of conventional optimization techniques. They therefore all inherit well known problems plaguing these conventional optimization methods, namely, poor convergence and susceptibility to getting trapped in local extremum. If the starting point of the algorithm is not well chosen, the solution can diverge, or get trapped at a local minimum. This is especially true if the objective function landscape contains isolated valleys or broken ergodicity.
In this paper we introduced a cost function based on the spatial geometry principles. In this method Nonlinear Programming is used as the optimization tool. The innovation of the proposed method is the estimation simultaneity of all intrinsic camera parameters using the cost function that includes all intrinsic camera parameters and normalized projected points of the real world. Result shows an accurate and feasible estimation of intrinsic camera parameters.

II. Nonlinear Programming

Nonlinear programming (NLP) is the process of determining unknown parameters of system for minimizing an objective function subject to nonlinear system equations and some equal or non equal conditions. The NLP problems have the common following form:

$$\min_{x \epsilon X} f(x)$$
$$s.t.\ g(x) \leq 0$$

$$X \subseteq R^{n}$$

where X is a subset of R^{n_x}, and f : x → R, g: X → R^{n_g} and f is the *objective function*.g_i(x) ≤ 0, i = 1,..., n_g, are *inequality constraints*, and h_i(x) = 0, i = 1,……., n_h,are *constraints*. Note also that the set X typically includes lower and upper bounds on the variables; the reason for separating variable bounds from the other inequality constraints is that they can play a useful role in some algorithms, i.e., they are handled in a specific way.

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Easily we can say that the NLP is the problem of finding a feasible point x^* such that $f(x) \geq f(x^*)$ where x^* is a feasible solution, which satisfies all constraints. for each feasible point x needless to say, a NLP problem can be stated as a minimization problem, and the inequality constraints can be written in the form can be written in the form $g(x) \geq 0$.

III. Camera Model

Earlier camera calibration techniques usually employed a perfect pinhole camera model.

$\boldsymbol{p_w}$: A point in a three-dimensional (3D) space in front of the camera.

$$P_w = \begin{bmatrix} P_{wx} \\ P_{wy} \\ P_{wz} \end{bmatrix} \quad (1)$$

p_c: Coordinates of p_w in camera system coordinates.

$$P_c = \begin{bmatrix} P_{cx} \\ P_{cy} \\ P_{cz} \end{bmatrix} \quad (2)$$

x^* ,y^* are the coordinates of the principal point in the
Image plane:

$$\begin{bmatrix} x^* \\ y^* \end{bmatrix} = \frac{f}{P_{cz}} \begin{bmatrix} P_{cx} \\ P_{cy} \end{bmatrix} \quad (3)$$

The relation between the real world coordinate (3D coordinate) and camera system coordinate is as follows:

$$P_w = M_{cw} P_C + e , \quad M_{cw} = \begin{bmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{bmatrix} \quad (4)$$

$$P_C = M_{wc} (P_w - e) , \quad M_{cw}{}^T = M_{wc} \quad (5)$$

Where
$\boldsymbol{e}$: canonical center coordinate
$\boldsymbol{M_{cw}}$: Camera orientation

IV. Marker projection on the Image Plane Procedure

We assume $\boldsymbol{m}$ is a point in the camera system coordinate

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$$\begin{bmatrix} x_m \\ x_m \\ x_m \end{bmatrix} \quad (6)$$

n is the normalized coordinates of ***m*** based on pinhole camera model

$$\begin{bmatrix} x_n \\ y_n \end{bmatrix} = \begin{bmatrix} x_m/z_m \\ y_m/z_m \end{bmatrix} \quad (7)$$

r is defined as follows:

$$r^2 = x_n{}^2 + y_n{}^2 \quad (8)$$

By applying lens distortion model to ***n***,
d is obtained.

$$\begin{bmatrix} x_d \\ y_d \end{bmatrix} = (1 + k_1\, r^2 + k_2\, r^4\,) \begin{bmatrix} x_n \\ y_n \end{bmatrix} \quad (9)$$

After applying lens distortion model by applying camera model matrix, the final pixel position related to the projection of ***m*** in camera image plane, calculated as follows:

$$\begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & \alpha f_x & x_R \\ 0 & f_y & y_R \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_d \\ y_d \\ 1 \end{bmatrix} \quad (10)$$

The above square matrix (K) is called Calibration Matrix.
The constraints are obtained from equations (11) and (12).

$$f_x.x_n.\,(1+k_1.r^2+k_2.r^4) + x_R\ \text{-}x_p=0 \quad (11)$$

$$f_y.y_n.\,(1+k_1.r^2+k_2.r^4) + y_R\ \text{-}y_p=0 \quad (12)$$

V. Objective Function

Earlier than introducing the objective function it is necessary to explain the concept of vanishing point comprehensibly.

A. Vanishing point of a line

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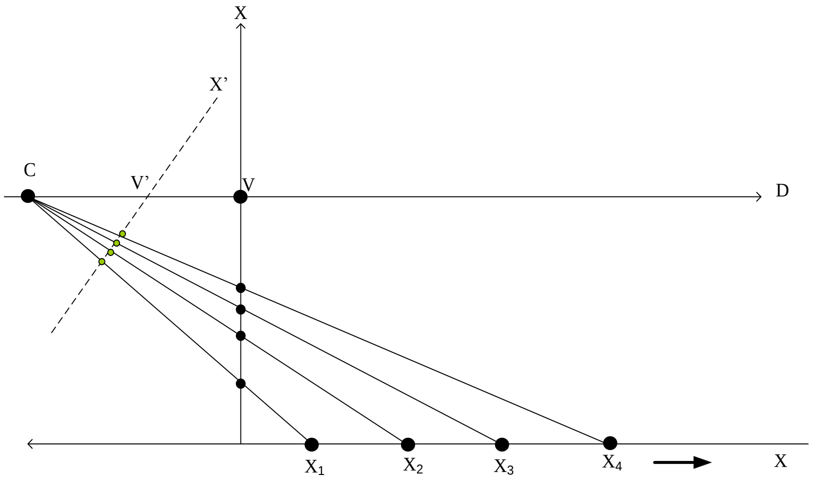
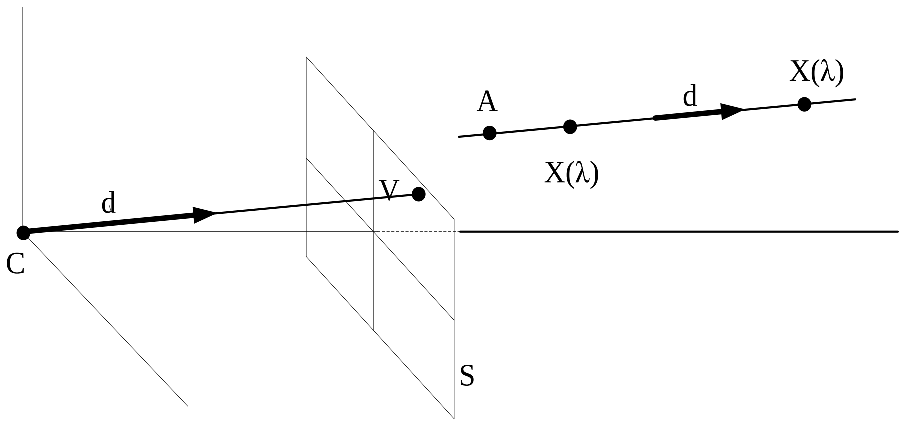
The vanishing point of a line is a point on the image plane that is obtained by intersecting the image plane with a ray parallel to the world line and passing through the camera centre.

The perspective geometry that gives rise to vanishing points is illustrated in Figure 1. In this figure, the point C is the camera centre and plane S is the image plane. Vector d is a vector with direction of the line D. The vanishing point, v, of the line D with direction d, is the intersection of the image plane (plane S) with a ray parallel to d through C (camera centre). Thus a vanishing point of a line depends only on the direction of the line, not on its position. Consequently a set of parallel lines have a unique vanishing point, as illustrated in Figure 1. For example image of infinite railway lines, are converging lines, and their image intersection is the vanishing point for the direction of the railway. Therefore parallel world lines are projected as converging lines, and their image intersection is the vanishing point of them.



(a)



(b)

Figure. 1. Vanishing point formation**,** *(a) Plane to line camera. The points* X_i , i= 1,... ,4 *are equally spaced on the world line, but their spacing on the image line monotonically decreases. In the limit*

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X $\rightarrow \infty$ *the world point is imaged at* x = v *on the vertical image line, and at* x' = v' *on the inclined image line. Thus the vanishing point of the world line is obtained by intersecting the image plane with a ray parallel to the world line through the camera centre* C. *(b) 3-space to plane camera. The vanishing point,* v, *of a line with direction* d *is the intersection of the image plane with a ray parallel to* d *through* C.

B. Objective function introduction

Consider the rectangle ABCD in 3D world and quadrilateral (A' B' C' D') as the image of rectangle ABCD on the image plane of the camera and suppose that this camera is calibrated perfectly. As shown in the fig. 1, lines AB and CD are parallel; therefore their image intersection (point s) is vanishing point of AB and CD. Also AD and BC are parallel and their image intersection (point t) is vanishing point of AD and BC. Consequently based on vanishing point definition, os is parallel with AB and CD and "ot" is parallel with AD and BC in 3D world. t and s are the vanishing points of orthogonal lines, therefore inner product of os and ot is zero.(figure 2)

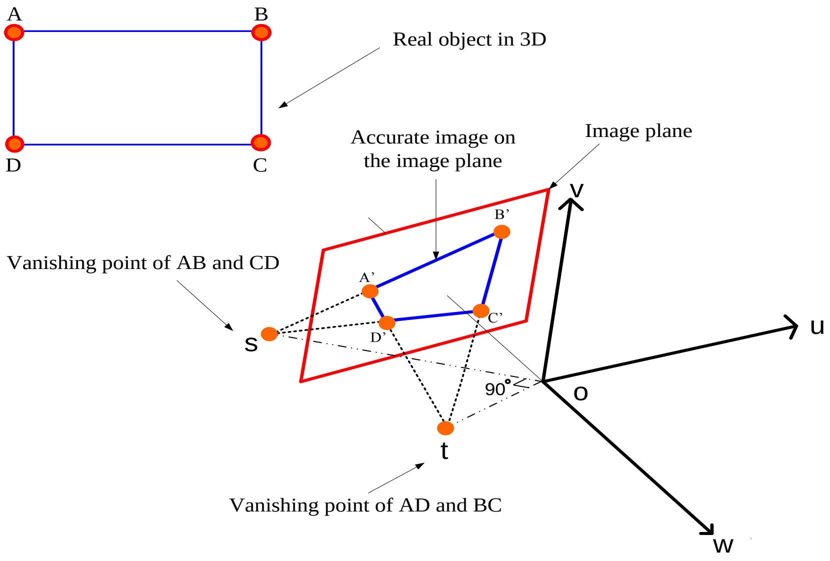

Figure. 2. Marker projections on image plane and the cost function based on spatial geometry rules

Assume camera is not calibrated correctly and suppose that instead of correct image (A'B'C'D'), quadrilateral A"B"C"D" imaged on the image plane of the camera, so the intersection of A"B" and C"D" (point s') is not vanishing point of AB and CD ,s' and s aren't coincident certainly. Also AD and BC are parallel and their image intersection (point t' that is the intersection of A"D" and C"B") is not vanishing point of AD and BC t' and t aren't coincident certainly. So os' and ot' lines isn't orthogonal therefore inner product os' by ot' isn't zero. (figure 3)

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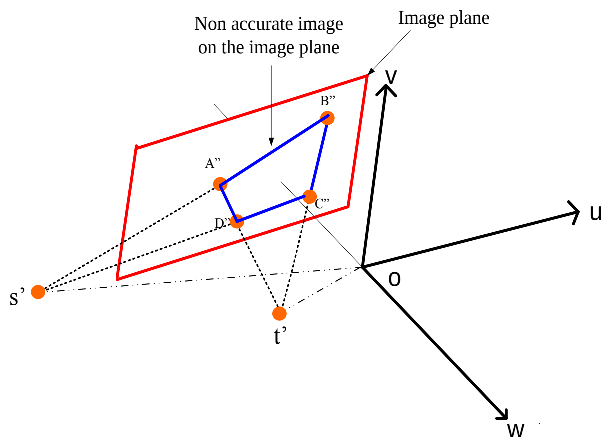

Figure. 3. Marker projections on image plane and the cost function based on spatial geometry rules

In this paper inner product is defined as objective function, if camera is calibrated correct inner product os by ot, which must be minimized. It's obvious that if the camera is calibrated properly, s' and t' are coincident on s and t respectively, therefore inner product which must be minimized.

VI. Experimental Results

According to the information provided by the camera manufacturer, values of the intrinsic camera parameters are as follows: $f_x = f_y$=720, x_R = 320, y_R = 240, α = 0 and $k_1 = k_2$= 0.

$(f_x,\ f_y)$ focal length, $(x_R,\ y_R)$ Principal point coordinate, $(k_1,\ k_2)$ lens distortion and α is skew coefficient that here we assume it is equal to zero.

With assumption of knowing the intrinsic parameters, we can calculate the position of points on the image plane. (x_p, y_p)
Based on the inverted camera model (k^{-1}), x_d and y_d are estimated.

$$\begin{bmatrix} x_d \\ y_d \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{f_x} & 0 & -\frac{x_R}{f_x} \\ 0 & \frac{1}{f_y} & -\frac{y_R}{f_y} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} \qquad (13)$$

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According to equation (14) related to lens distortion parameters, the equation between distorted points (x_d, y_d) and normalized points (x_n, y_n) corresponding to the projection of the markers on the image plane is as follows:

$$\begin{bmatrix} x_n \\ y_n \end{bmatrix} = \frac{1}{(1+ k_1 r^2 + k_2 r^4)} \begin{bmatrix} x_d \\ y_d \end{bmatrix} \qquad (14)$$

Where equation (8) shows the relation between (x_n, y_n) and $\boldsymbol{r}$ in the above equation.
The objective function depends on a set of (x_n, y_n) on the image plane that are calculated from equation (14), and also other intrinsic camera parameters. Minimizing this objective function by Nonlinear Programming results in an accurate intrinsic camera parameters estimation.

Table I shows the results of the estimation by Nonlinear Programming based on various noise values, where k_1 and k_2 are constant and $k_1 = k_2 = 0$.

TABLE I: estimation results by Nonlinear Programming based on various noise values, where k_1 and k_2 are constant and $k_1 = k_2 = 0$.

Noise level	*fx*	*fy*	*XR*	*YR*
0	719.9556	721.0308	319.0014	240.6515
0.05	722.545	723.922	321.084	238.7119
0.1	716.8452	729.652	318.9502	239.3665
0.15	710.225	717.5412	326.379	246.6462
0.2	709.3547	712.5698	329.5421	245.2248

Table II, shows the results of the estimation by Nonlinear Programming based on various noise values, where k_1 and k_2 are constant and $k_1 = 0.01$, $k_2 = 0.001$.

TABLE II: estimation results by Nonlinear Programming based on various noise values, where k_1 and k_2 are constant and $k_1 = 0.01$, $k_2 = 0.001$

Noise level	*fx*	*fy*	*XR*	*YR*
0	718.5489	724.5625	317.895	243.784
0.05	717.2259	727.5487	326.8457	245.5694
0.1	710.561	718.548	315.6511	236.455
0.15	710.668	707.894	315.6874	234.8875
0.2	709.5414	710.5796	310.5833	231.8541

Table III shows the results of the estimation by Nonlinear Programming based on various noise values, where k_1 and k_2 are variables.This means that lens distortion is considered here.

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TABLE III: estimation results by Nonlinear Programming based on various noises values, where k_1 and k_2 are variables. This means that lens distortion is considered here.

Pe	*fx*	*fy*	*XR*	*YR*	*K1*	*K2*
0	718.0318	723.248	320.8841	241.3108	-3165e-7	631e-8
0.05	722.9632	725.1154	319.0064	241.0171	6353e-7	5032e-8
0.1	724.4826	724.8421	317.2564	238.3443	117e-8	165e-9
0.15	729.2331	715.7809	325.1152	245.1457	365e-7	7528e-8
0.2	728.907	729.1426	310.5887	246.2149	713e-7	5064e-8

VII. Conclusion

The cost function is obtained by vanishing Point Theorem which is one of the spatial geometry principles also Nonlinear programming is used for minimizing the objective function. Calibration on a sample camera based on the proposed objective function resulted an accurate and feasible estimation of intrinsic camera parameters.

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