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Research Article

The Relationship between Entropy and Entanglement of Bipartite System

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Abstract
In this work, we first study arrangement of bipartite system using linear entropy, and effect parameters: magnetic field, temperature, spin-spin interaction and Dzyaloshinski-Moriya interaction. Then we study and comprised entanglement of system, using concurrence and negativity measures. Also, we show that the increase in entropy of the system is reduced to tangled systems.

Keywords: Thermal entanglement, Linear entropy, density matrix, spin chain, concurrence, negativity

I. Introduction

Entangled state is a state which cannot be shown based on tensor product of individual states of comprising subsystems. Two systems are called entangled when experimenting on one system can reveal information about the other one. This nonlocal quantum characteristic will make us able to do some tasks (e. g. quantum teleportation, quantum cryptography and quantum dens coding) which are impossible to do without it (M. A. N sen and I. L. Chuang, 2000; G. Beneti et. al, 2004). according to Boltsman principle, a system with Hamiltonian of H, and thermal equilibrium with probability of $\frac{1}{Z} exp\left(\frac{-E_i}{k_B T}\right)$ is state of $|\varphi_i\rangle$, which is eigenvector corresponding to Eigen value E_i. Density matrix of such systems is determined by

$$\rho(T) = \frac{1}{Z} exp\left(\frac{-H}{k_B T}\right) = \frac{1}{Z} \sum_{i=1}^{N} exp\left(\frac{-E_i}{k_B T}\right) |\varphi_i\rangle\langle\varphi_i| \qquad (1)$$

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Where T is the temperature, k_B Boltsman constant and Z partition function of the system. Entanglement obtained from this density matrix is called thermal entanglement. Many studies have been done, For example, Thermal entanglement versus mixture in a spin chain(Ch. Xian, Ch. Zhi, X. Min and G. Can, 2006) Thermal entanglement of spin in an inhomogeneous magnetic field(M. Asoudeh and V. Karimipour, 2005) Teleportation and thermal entanglement in two qubit Heisenberg XYZ spin chain with the DM interaction and the inhomogeneous magnetic field(D. Gao, Sh. Zhaog, D. Zhu and F. Wang Hong, 2010).

In this article we study entropy of the two qubit spin chain with DM interaction and the external magnetic field in the Z axis direction. Also we determine entanglement using concurrence and negativity measures. Finally, we discus about entropy and entanglement of system.

II. Linear entropy

Because of decoherent, a pure state change to a mixed state, but in the more quantum information processing, is necessary state with a high purity and amount maximally entangled. Therefore, it is necessary to know the relationship between the mixture and entanglement of system. Entropy is a measure of the regulatory of system. Which using liner entropy, express as follows(Ch. Xian, Ch. Zhi, X. Min and G. Can, 2006),

$$S_L = \frac{N}{N-1}\left[1 - \mathrm{Tr}[\rho(T)^2]\right] \qquad (2)$$

The entropy range from zero to one, zero value is a completely mixed state and one value for absolutely pure state. N show that dimensional of Hilbert space, the linear entropy of the system is as follows:

$$S_L = \frac{4}{3}\left[1 - \mathrm{Tr}[\rho(T)^2]\right] \qquad (3)$$

The Hamiltonian for two qubit spin chain can be expressed as,

$$\hat{H} = \frac{J}{4}(\sigma_1^X\sigma_2^X + \sigma_1^Y\sigma_2^Y) + \frac{(B+b)}{2}\sigma_1^Z + \frac{(B-b)}{2}\sigma_2^Z + \frac{D}{4}(\sigma_1^X\sigma_2^Y - \sigma_1^Y\sigma_2^X) \qquad (4)$$

Where J and D are spin-spin and spin- orbit interactions. b and $\hat{\sigma}_i$ are the inhomogeneous of magnetic field and Pauli matrix, respectively. We can obtain eigenvalues and iegenvector for the above Hamiltonian as follows:

$$E_1 = -B, \quad E_2 = B, \quad E_3 = -\frac{1}{2}\sqrt{(4b^2 + D^2) + J^2}, \; E_4 = \frac{1}{2}\sqrt{(4b^2 + D^2) + J^2}$$

$$\psi_1 = |1\,1\rangle, \psi_2 = |0\,0\rangle, \psi_3 = C(N|0\,1\rangle + |1\,0\rangle), \psi_4 = L(P|0\,1\rangle + |1\,0\rangle)$$

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L, C, N and P parameters in the above equations are,

$$N = -\frac{i(-4b+2\sqrt{(4b^2+D^2)+J^2})}{2D+i2J} \;,\; C = \frac{1}{\sqrt{N^2+1}}, P = \frac{i(4b+2\sqrt{(4b^2+D^2)+J^2})}{2D+i2J} \;,\; L = \frac{1}{\sqrt{P^2+1}}$$

Inserting eigenvalues and eigenvectirs in Eq. (1), we can obtain density matrix,

$$\rho(T) = \frac{1}{Z}\begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & \xi & \eta & 0 \\ 0 & \eta^* & \upsilon & 0 \\ 0 & 0 & 0 & \gamma \end{pmatrix} \qquad (5)$$

For our investigation, we assume that Boltsman and Planck constants is equal with one. In the above equation Z is partition function, which can be written as follows,

$$Z = \exp\left(\frac{-E_1}{KT}\right) + \exp\left(\frac{-E_2}{KT}\right) + \exp\left(\frac{-E_3}{KT}\right) + \exp\left(\frac{-E_4}{KT}\right)$$

The density matrix elements is,

$$\mu = \exp\left(\frac{-E_2}{KT}\right), \gamma = \exp\left(\frac{-E_1}{KT}\right), \eta = NC^2\exp\left(\frac{-E_3}{KT}\right) + PL^2\exp\left(\frac{-E_4}{KT}\right)$$

$$\xi = N^2C^2\exp\left(\frac{-E_3}{KT}\right) + P^2L^2\exp\left(\frac{-E_3}{KT}\right), \nu = C^2\exp\left(\frac{-E_3}{KT}\right) + L^2\exp\left(\frac{-E_4}{KT}\right)$$

Using equation (3) can be a qualitatively investigated mixture system. In fig.1 linear entropy versus temperature for different DM is plotted. The figure show that by increasing the temperature, the linear entropy rise and mixture of system at the zero temperature is minimal. It also enhances the spin-orbit interaction, the amount of mixture of system decreases at the zero temperature, In other words, with enhances spin-orbit interaction, order of system increasing.

In Fig.2 linear entropy versus temperature for different magnetic fields is plotted. It is clear that by increasing the external magnetic field, the area under the curve decreases, namely; the order of the system increases.

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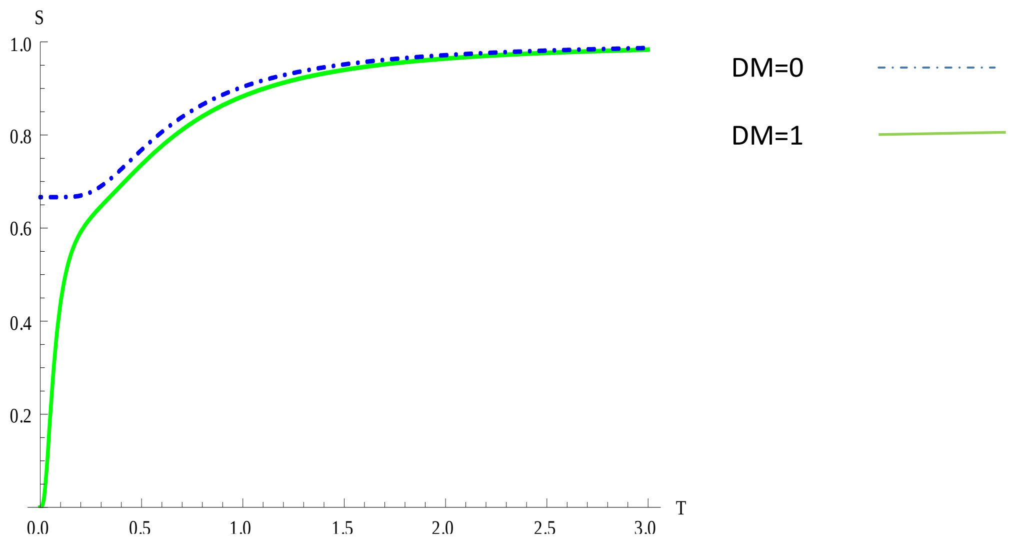

Fig.1. The linear entropy versus temperature for different DM and B=1, J=2, b=0

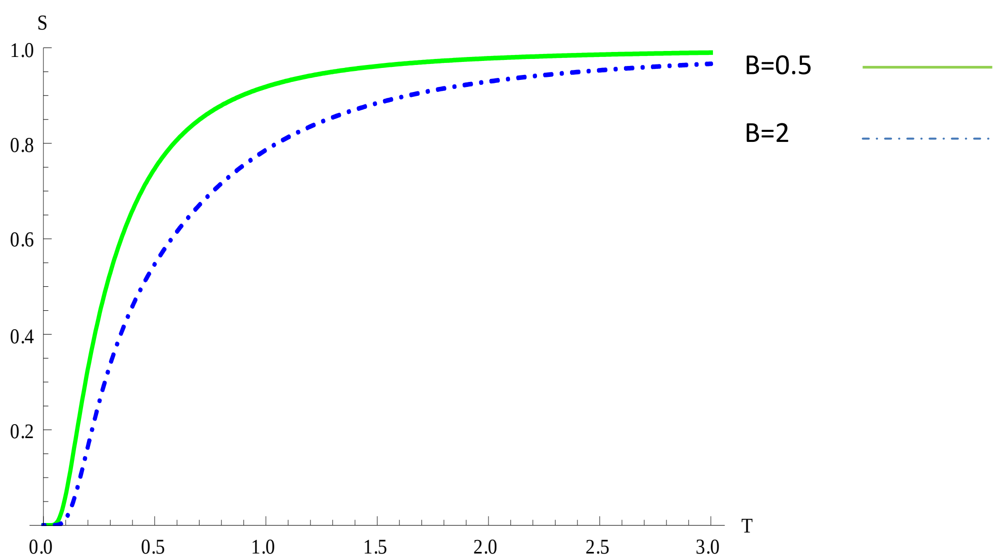

Fig.2. The linear entropy versus temperature for different magnetic fields and D=1, J=2, b=0

In Fig.3 demonstrate linear entropy versus inhomogeneous magnetic field parameter for different spin-spin interaction. It shows that by increasing parameter b, the amount of linear entropy decreases, namely; the order of the system increases. It also clear that by increasing the spin-

spin interaction, the amount of mixture of system decreases, namely; order to strengthen the system.

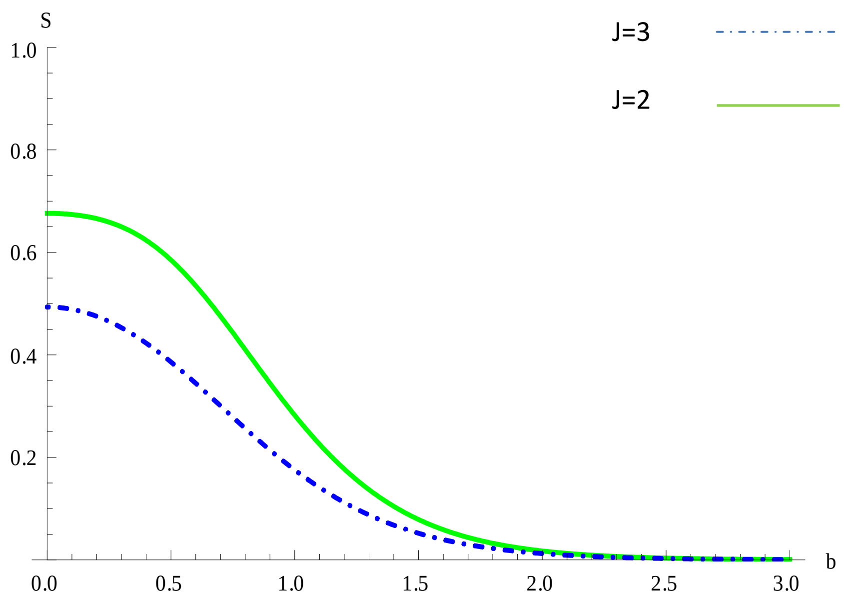

Fig.3. The linear entropy versus inhomogeneous magnetic field parameter for different spin-spin interaction and D=1, B=1, T=0.5

III. Comparison concurrence and negativity measures

The concurrence for each density matrix in form of Eq.5. is (W. K. Wootters, 2001),

$$C(\hat{\rho})=\frac{2}{Z}\max\{0,|\eta^*|-\sqrt{\xi\upsilon}\} \tag{6}$$

System entanglement can be analyzed qualitatively with regard to preceding concurrence function. In fig.4 concurrence versus temperature for different magnetic fields is plotted. It is clear that by increasing the external magnetic field, interval of entanglement decreases. Also, increase in temperature leads to a decrease of tangled.

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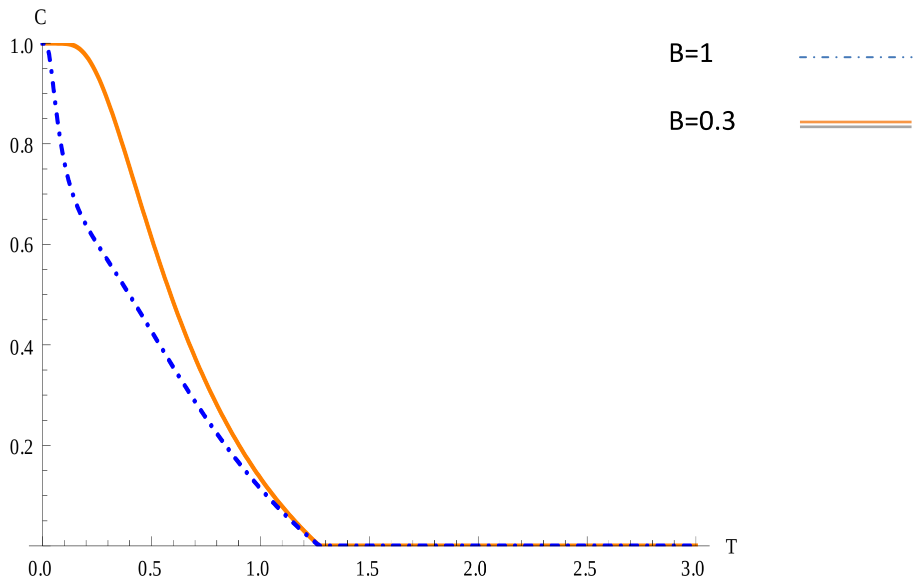

Fig.4. The concurrence versus temperature for different magnetic fields and b=0, D=1 , J=2

The negativity for each bipartite system is (G. Vidal, R.F. Werner,2002),

$$N(\rho)=\frac{\|\rho^{T_A}\|-1}{2}=\sum_i|\mu_i| \qquad (7)$$

Which ρ^{T_A} is the partial transpose and μ_i, eigenvalues are negative as to ρ^{T_A}. To obtain the partial transpose and inserting the above equation, we obtain negativity function. Fig.5 shows that by increasing the external magnetic field and temperature, the amount of entanglement decreases. Also, to comparing Fig.4 and fig.5 it is known that the maximum entanglement from negativity, is half the maximum value obtained concurrence.

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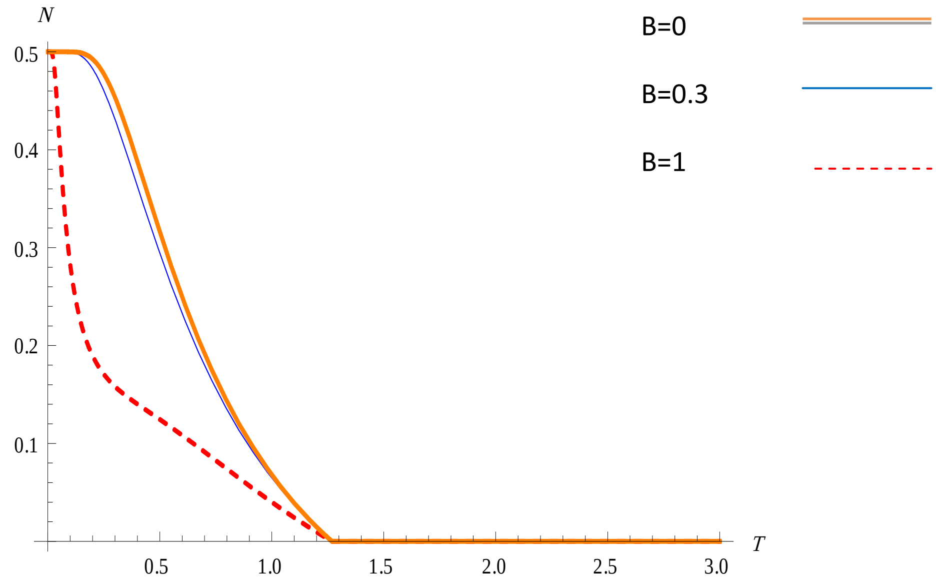

Fig.5.The negativity versus temperature for different magnetic fields and b=0, D=1 , J=2

IV. Conclusions

To study entropy and entanglement of systems, we obtain results as follows,

- Increasing the temperature, leads to development entropy of system.
- With increasing spin- spin and spin- orbit interactions and external magnetic field, mixture (entropy) of system decreases.
- With increasing spin- spin and spin- orbit interactions and external magnetic field, entanglement of system decreases.
- Increasing in temperature, leads to growing entanglement from negativity and concurrence.
- Growing the inhomogeneous magnetic field parameter, leads to entanglement and critical temperature, decrease and rise, respectively.
- Maximum entanglement from negativity, is half the maximum value obtained concurrence.
- Relationship between entropy and entanglement is an inverse relation, namely; entanglement is kind of order.
- Maybe each of entanglement measures, are represents one of its applications, for example, concurrence a good represents for quantum transfer of information.

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