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## Research Article

# Numerical computation of the seepage Rate under the Hydraulic structures (case study on the Dez Regulating Dam in Khuzestan, Iran)

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### Abstract

Hydraulic structures which contain surface water, control floods and create storage reservoirs are designed for optimum water use. The stored water, due to high potential hydraulic head seeps through the porous foundation of the structures towards downstream. Seepage is one of the most important design challenges for small and large dams. Regardless of the type of dam. One can say that much of the activities involved for design construction and operation activities and a significant part of the costs for design, construction and operation of water control structures is related to seepage control.

Measured the seepage rate for stability analysis of hydraulic structure under various hydraulic conditions is of high necessity. Principles of seepage calculation are based on flow equations on porous environment. This investigation uses finite elements method with three nodes triangular elements for numerical differential equation solving steady flow and drawing flow network downstream of the hydraulic structures. This is carried out under porous environmental conditions in different parts in order to calculate the hydraulic head pressure as well as seepage rate under the hydraulic structures in question.

The main focus of investigation is on the foundation of Dez regulating dam which has incorporated cut-off wall and impervious blanket zones. Results indicate that the proposed model was able to calculate the exact rate of seepage flow and flow network under the hydraulic structures in questions.

**Key words:** seepage, finite elements, meshing, flow net

### 1. Introduction:

In engineering terms, water movement in porous environments is called seepage that happens through hydrostatic pressure between two point from porous environment.

Seepage of water from foundation soil, causes pressure on bottomland tendency of soil to be washed. These effects of seepage from soil under structures often cause serious damage to them. Seepage issues in hydraulic structures that happen in permeable soils, are one of the determining factors of final design and dimensions of these structures.

Soil is collection of solid particles that there are pores in it in away that water can flow from a point with high potential to a point with less potential.

Principles of seepage's calculations are according to Laplace's equation and using of flow net. For drawing of flow net and calculating of seepage's debye, numeral solution of differential equation of continuity ( Laplace ) is needed. Numeral methods have been very useful in solving of seepage's questions (because of complicated equations and difficulty of study these kinds of questions with analytical methods) and now with computers' development, they have gain very high efficiency.

Method of limited parts is one of the numeral methods for solving of differential equations that in addition of simplicity of calculations, has high capability in solving complicated questions and presents acceptable results.

In this article, differential equation in question is extracted and after that a computer program is in traduced for modeling of that equation.

## **2. Relations of water's flow in soil:**

According to Bernoulli's relation, total head of a water point while flowing is equal to pressure's load plus velocity's load plus height's load. It means:

$$h = \frac{P}{\gamma_w} + \frac{v^2}{2g} + Z \quad (1-2)$$

In which, h: total head, p: pressure, v: velocity, g: gradient of gravity,  $\gamma_w$ : weight for water. Velocity's load can be ignored because of low speed of water's flow in soil and total head can be written in this way:

$$h = \frac{P}{\gamma_w} + Z \quad (2-2)$$

Drop (waste) of charge between a ,b points (in soil) is written in this way:

$$\Delta h = h_A - h_B = \left( \frac{P_A}{\gamma_w} + Z_A \right) - \left( \frac{P_B}{\gamma_w} + Z_B \right) \quad (3-2)$$

Drop of charge h can be written in dimensionless from:

$$i = \frac{\Delta h}{L} \quad (4-2)$$

$i$ : hydraulic gradient and  $L$ : distance between A and B points. In other words, length of flow in which the drop of charge occurs.

### **1-2- Darcy's law**

Permeability is one of the properties of porous environment that shows flow rate of fluid from non-continuous and empty spaces like soil layers. Flow of seepage can be synchronous or non synchronous, under pressure and have a free surface.

For example, flow under a weight dam or reservoir passing from an earth dam can be named. Although the erotically all soils are porous, in practice word of Permeability or permeable is applied to the soils in which water can flow easily.

The soils in which water hardly passes are called permeable. For understanding the process of water penetration, lots of experiments have been performed that Darcy's experiences are very important in them .

Darcy did his experiment on a tube, which was full of sand. from his observations about flow through horizontal layers of sand, it was concluded that intensity of flow has direct ratio with pressure drop and inverse ratio with length of flow's path and also is compatible with permeability coefficient,  $k$ , which is depended on sand's nature.

Darcy's law can be written in this way:

$$Q = KA \frac{(h_A - h_g)}{L} \quad (5-2)$$

$$Q = KA i \quad (6-2)$$

$$h = z + \frac{p}{\gamma} \quad (7-2)$$

$i$  is hydraulic gradient and apparent velocity  $V$ , that is called special debye and is:

$$V = \frac{Q}{A} = ki \quad (8-2)$$

Although Darcy's law is indicated as a completely empiric relation and is result of his laboratorial observations, but scholars have proved this relation the erotically.

For example, Darcy's relation is extracted on basis of forces on a water-soil system.

Also, Hille-shaw acquired this relation from Navir stokes's equations. Darcy's law can be written in three directions of X,Y,Z:

$$V_x = -K_x \frac{\partial H}{\partial x} \quad (9-2)$$

$$V_y = -K_y \frac{\partial H}{\partial y} \quad (10-2)$$

$$V_z = -K_z \frac{\partial H}{\partial z} \quad (11-2)$$

In these relations,  $K_x$  and  $K_y$  and  $K_z$  indicate hydraulic conductivity coefficient in X,Y,Z directions, also  $V_x$  and  $V_y$  and  $V_z$  indicate velocity of flow in X,Y,Z direction.

### 2-2- coefficient of permeability

Coefficient of permeability is one of the important factors in analysis of water's permeation in layers of soil and it has the same measure of flow's velocity. However, coefficient of permeability often is used by geotechnique engineers.

Geologists call it hydraulic conductivity. In units systems SI, coefficient of permeability is according to  $\frac{cm}{s}$  (centimeter on second) or meter in day. Coefficient of permeability of soil depends on several factors like:

Concentration of fluid, amount and distribution of porosity, aggregation curve, porosity ratio, coarseness of grain's surface and degree of soil's saturation.

### 3-2- Laplac's equation:

For analysis of how water seeps in layers of soil, a general model for describing seepage is needed. With determining of boundary conditions and soil properties, this model can be used for determining of charge and distribution of flow and amount of seepage.

Laplace's equation is a mathematical basis for different models or methods which are used in analysis of seepage. Laplace's equation is for water's flow in soil. Of course, for using Laplace's equation in porous environment, these suppositions are needed:

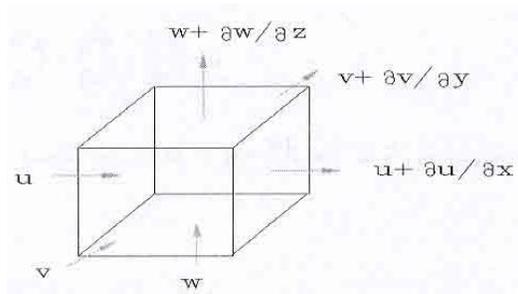
1. Soil is homogenous
2. Empty spaces of soil are completely full of water. (Saturated environment)
3. There is no soil's inflation or compression.
4. Flow is layering in a way that Darcy's law is considered.

With this presupposition that if flow is steady and there is no change in mass of soil and water's element, amount of input water to element is equal to output water from that.

If velocity in X,Y,Z direction is considered W,V,U respectively and hydraulic gradient is  $I_z, I_x, I_y$ , according to figure 2-1, amount of input water to the cubic element is equal to  $Udx dy + Vdx dz + Wdx dy$ .

With these equations, amount of output water from elements can be calculated in this way:

$$Q_w = \left( u + \frac{\partial u}{\partial x} dx \right) dz dy + \left( v + \frac{\partial v}{\partial y} dy \right) dx dz + \left( w + \frac{\partial w}{\partial z} dz \right) dx dy \quad (12-2)$$



Equality of amount of input and output water makes equation of continuity as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (13-2)$$

Now, if  $K_x, K_y, K_z$  be permeability amounts in triple directions and  $h$  be element's charge, with Darcy's law we can have:

$$\begin{aligned} u &= -k_x \frac{\partial h}{\partial x} \\ v &= -k_y \frac{\partial h}{\partial y} \\ w &= -k_z \frac{\partial h}{\partial z} \end{aligned} \quad (14-2)$$

Then:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -k_x \frac{\partial^2 h}{\partial x^2} - k_y \frac{\partial^2 h}{\partial y^2} - k_z \frac{\partial^2 h}{\partial z^2} \quad (15-2)$$

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \quad (16-2)$$

If the soil be homogenous  $K_x=K_y=K_z$ , we will have:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (17-2)$$

For isotope soils, function  $\phi = -kh$  which is called function of flow's potential can be applied. So:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (18-2)$$

Gradient  $\phi$  in every direction, gives velocity in the same direction in this way:

$$\frac{\partial \phi}{\partial x} = -k \frac{\partial h}{\partial x} = u \quad (19-2)$$

If flow is supposed to be two dimensional (only x,y directions ) equation 18-2 becomes simpler.

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (20-2)$$

Equation 18-2 indicates two identical curves that cut each other with right angle and form a squared network. This figure is called flow's network.

### 3. Finite elements method

There are two general methods in solving questions with finite elements method:

1. Changing method
2. Method of weight remainders (Gallerkane method)

Since method of weight remainders is more common is hydraulic engineering and also because of it's high usage in analysis of seepage and moving of underground waters, it is used as an analytical tool.

In this method, a primary approximation is defined for boundary or primary conditions. This method makes weight average of remainders in each point of limited element network equal to zero.

Each equation can be written in this way:

$$L(h(x, y, z)) - F(x, y, z) = 0 \quad (1-3)$$

In which, L is a differential operator and h(x,y,z) is dependant variable and F(x,y,z) is a known function that doesn't include dependant variable.

$$\hat{h}(x, y, z) = \sum_{i=1}^m N_i(x, y, z) h_i \quad (2-3)$$

In this relation, Ni is inside function or function of figure and his are amount of dependant variable in each node.

Function of figure, is a simple formula for producing approximate amount in each point (x,y,z) on the basis of amounts of node points. The important property of functions of figure is that in the related node, they will have one amount and in other nodes, they will be zero. Since h(x,y,z) is an approximation, differential equation doesn't completely satisfy and there will be a remainder that is written as 3-3 relation:

$$L(h(x, y, z)) - F(x, y, z) = 0 \quad (3-3)$$

Generally, remainders are not zero and change from a point to the other one. In method of weight remainders, we impose that weight average of all amounts of nodes' remainders come close to zero.

In this way, it should be:

$$\iiint_{\Omega} w(x, y, z).R(x, y, z).dx.dy.dz = 0 \quad (4-3)$$

The term w(x,y,z) is a weight function and  $\Omega$  is the question which should be integrated. With remainder and also with using of sentences of function of approximation, we will have:

$$\iiint_{\Omega} w(x, y, z).[L(\phi(x, y, z)) - F(x, y, z)].dx.dy.dz = 0$$

### 1-3- Finite elements method

In analysis of Darcy's flow as it was said equations of limited components of seepage in Darcy's situation can be gained by change method or Gallerkane method, in this part we present Gallerkane method.

This method includes:

First step: v zone (total zone) is divided to E limited components that each of them has P nodes.

Second step: A proper figure of h's changes in elements is supposed and  $h^e(x,y,z)$  in element e is said in this way:

$$h^{(e)}(x, y, z, t) = [N(x, y, z)]^{-(e)} h \quad (5-3)$$

In which N is function of figure.

Third step: in Gallerkane method, weight remainder on element zero is applied in this way:

$$\iiint_{V^{(e)}} N_i \left[ \frac{\partial}{\partial x} \left( k_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial h}{\partial z} \right) - c \frac{\partial h}{\partial t} \right] dV = 0 \quad i = 1, 2, \dots, p \quad (6-3)$$

With Garien-Gaus theorem, the first sentence of this equation could be written in this way:

$$\iiint_{V^{(e)}} N_i \left[ \frac{\partial}{\partial x} \left( k_x \frac{\partial h}{\partial x} \right) \right] dV = - \iiint_{V^{(e)}} \frac{\partial N_i}{\partial x} k_x \frac{\partial h}{\partial x} dV + \iint_{s^{(e)}} N_i k_x \frac{\partial h}{\partial x} l_x .dS \quad (7-3)$$

In which  $L_x$  is conductor cosine of tension to outside perpendicular on surface in x direction, with similar relations for second and third Sentences of integral, we can write 6-3 equation as:

$$\iiint_{v(e)} \left[ k_x \cdot \frac{\partial N_i}{\partial x} \frac{\partial h}{\partial x} + k_y \cdot \frac{\partial N_i}{\partial y} \frac{\partial h}{\partial y} + k_z \cdot \frac{\partial N_i}{\partial z} \frac{\partial h}{\partial z} \right] dV + \iint_{s(e)} N_i \left[ k_x \cdot \frac{\partial h}{\partial x} l_x + k_y \cdot \frac{\partial h}{\partial y} l_y + k_z \cdot \frac{\partial h}{\partial z} l_z \right] dS + \iiint_{v(e)} N_i \left[ c \frac{\partial h}{\partial t} \right] dV = 0 \quad i = 1, 2, \dots, p$$

We have:

$$[k_1^{(e)}] \overline{h^{(e)}} + [k_2^{(e)}] \overline{h^{(e)}} + [k_3^{(e)}] \overline{h^{(e)}} = \vec{0} \quad (8-3)$$

In which matrixes  $[K_1^{(e)}]$ ,  $[K_2^{(e)}]$ ,  $[K_3^{(e)}]$  are:

$$[k_1^{(e)}] = \iiint_{v(e)} \left[ k_x \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + k_y \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} + k_z \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right] dV \quad (9-3)$$

$$[k_2^{(e)}] = \iint_{s(e)} h N_i N_j \quad (10-3)$$

$$[k_3^{(e)}] = \iiint_{v(e)} v N_i N_j .dV \quad (11-3)$$

These equations can be solved with applying boundary and primary conditions.

### 2-3- boundary conditions in Dez regulating dam (seepage from foundation of dam)

for better explaining of meshing (reticulation), the space beloved a concrete dam has been networked.

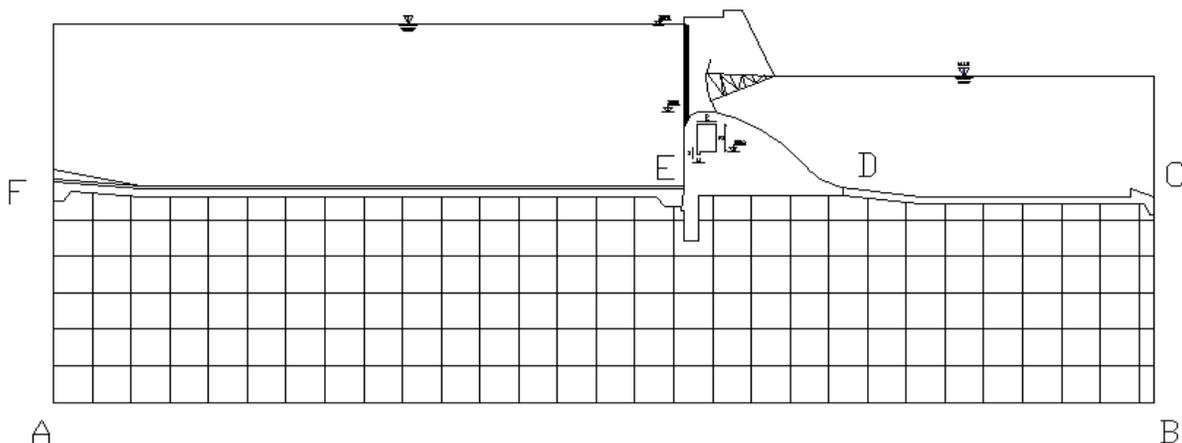


Figure. 1. Dez regulating dam

**A) Boundary of AB and AF**

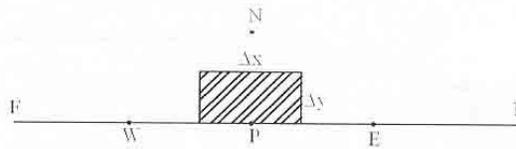
This boundary is a potential line and all of the points on this boundary have H1 potential.

**B) Boundary of CD and ED**

This boundary is a potential line and all of the points on this boundary have H2 potential.

**C) EF boundary (current)**

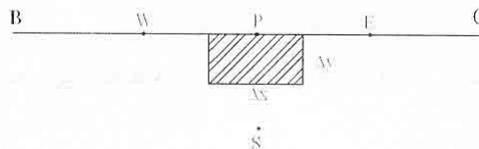
This boundary, is a flow line, it means there is no flow in perpendicular direction to the bottom:



**Figure. 2. EF boundary**

**D) BC boundary**

BC boundary is a flow line that there is no flow in perpendicular direction on it toward up.



**Figure. 3. BC boundary**

**4. Case study**

As it was said in abstract, the present model on the Dez regulatory dam is applied as an applicator case. Dez regulatory dam is constructed on the Dez river in the north of Dezfoul city. This dam is concrete and it has 140 meters length and 20 meters height and 6 sector valves. According to geology and geotechnique reports at the location of dam, foundation has permeable layer till 10 meters depth and after that it has conglomerate formation. In this study, amount of seepage in foundation has been considered till 20 meters depth. For decreasing of seepage from foundation of dam, a sealing wall with diameter of 1.5 meter and depth of 5 meters and also a clay lancet with length of 56 meters and diameter of 1 meter have been considered.



Figure. 4. Case study. Dez regulatory dam

## 5. Conclusion

In this study, equation of water's flow from under of water structures has been solved with finite elements method and using of weight remainder relation of Gallerkane.

The results from solving the equation are hydraulic charge in network that are calculated with using of passing Debye under structure. For this purpose, a computer program has been introduced and a regulatory concrete dam has been considered as a model.

Numeral results of this program in solving the seepage question under foundation of dam, show capability of model and appropriate performance of program.

### Properties of finite elements method in making network smaller in specific places:

Unlike finite volume method in which for making network smaller, all of mesh should be smaller and as result number of calculations and also time needed for analysis of model increases, with property of finite elements method in making network of meshing smaller, (only in specific parts) we can notably decrease number of calculations and necessary time of analysis.

In case study of this research, for increasing of accuracy's coefficient of model, a network with smaller meshing has been used in clay blanket and sealing wall place.

Of course it should be mentioned that unlike this public imagination that the smaller network, the more accuracy, this thought is only correct to a certain limit and after that with network becoming smaller, error increases and model deviates.

**Necessity of dividing foundation of structure to similar layers.**

It was concluded that for modeling foundation of water structures for control of seepage, it's better that foundation of structure divided into different zones that are similar in soil material and head of water and gradient of ground's layers and them amount of seepage of each of them be computed separately and for total debye of seepage under structure, all of the amounts will be added.

In a layered soil, in which coefficient of permeability for flow in a certain way, is different for various layers, determining an equivalent coefficient of permeability is needed.

**The effect of geology conditions of foundation on amount of seepage:**

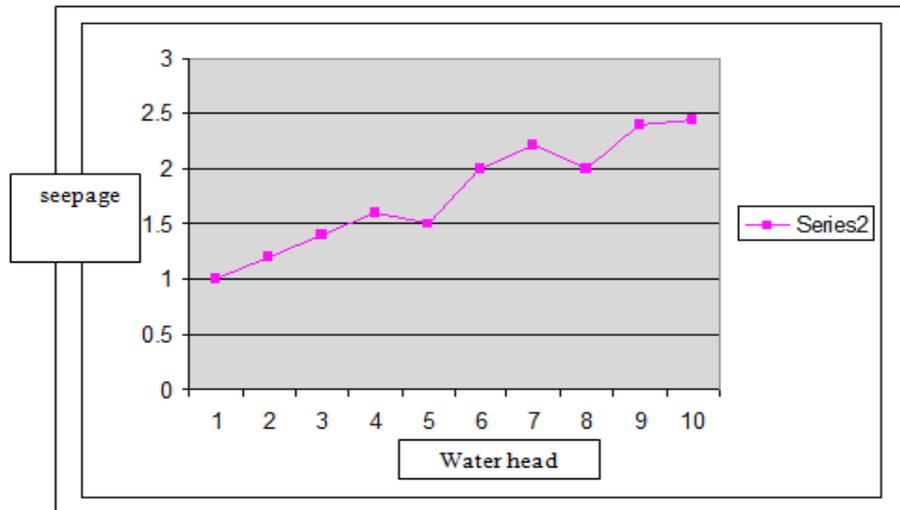
According to geology studies at Dez regulatory dam, existence of a conglomerate layer from kind of Bakhtiary stone has been confirmed.

With very low coefficient of permeability of this layer, amount of seepage under dam has decreased significantly and results of analysis of model approve it.

**The effect of amount of head of water on amount of seepage:**

In this research, a comparison had been performed for studding of amount of the effect of head of water behind the dam on amount of seepage under structure of dam. In this comparison, for each meter increase of water behind the dam, amount of seepage was calculated and registered and the results were shown in a diagram in this figure.

If water's level of up and down in different time paces be constant, the results of computer program will be similar to the results of steady condition.



**Figure. 5. effect of amount of head of water on amount of seepage**

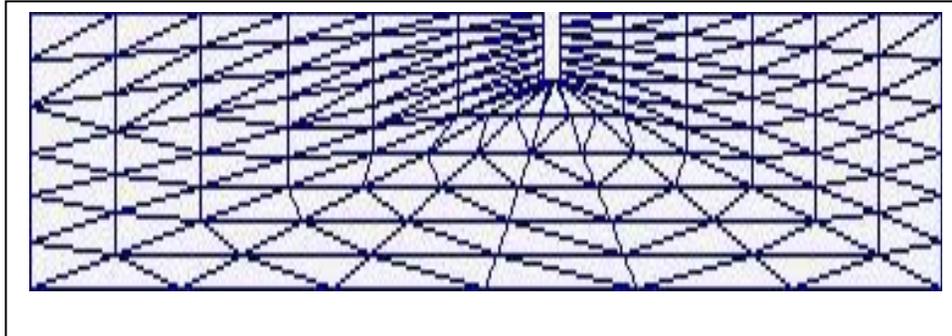
It was found that if foundation of structure has big grain stones, Darcy's flow is not true and non Darcy's flow should be used.

#### **Calculations and drawing of flow's network**

Initially dam body is introduced to the program. and place of clay blanket and sealing wall are determined for program. Foundation of dam in two layers is divided according to the geology studies and specifications of each layer are introduced.

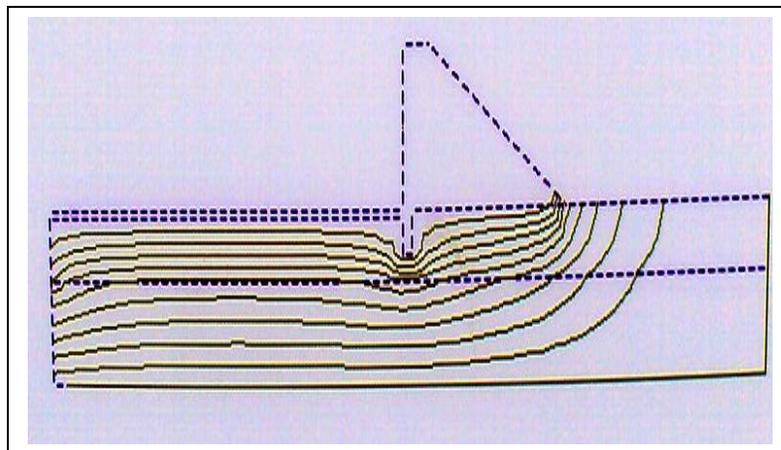
After that with determining of kind of program's meshing (three or four nodes), meshing will do foundation of the dam.

In this step, we can use introduced meshing program or we can introduce output of meshing to the program with a little changes.



**Figure. 6. Calculations and drawing of flow's network**

At the end, after analysis of model, program presents velocity in two directions and functions of potential in each point.



**Figure. 7. Analysis of model**

#### References:

1.N,Vahab Rajaie,1368. Solving seepage's question in dams using numeral methods of boundary element, limited elements and limited differences in volume of control, Mathis's of hydraulic structures, Tehran University.

2. Seid Alireza, Jahanara, 1387, analysis of earth dams under influence of dimensions of clay blankets with . Mathis's of hydraulic structures, Tehran university.
3. M. Hoseini and A. Sanati. 1378. Analysis of non liner flow of water from walls of gravel (rock fill) dam with finite elements method and in constant (steady) network frame. The fifth civil international conference. Mashhad Ferdosi university.
4. Mehrdad, Shahrbanouzadeh, 1382. Numeral model of 3D seepage from foundation of dams with method of limited volume. The sixth international conference of civil engineering.
5. Mohsen, Rashidi, 1387. Mathematical model of sheet seepage from body and foundation of dams, MA thesis.
6. A. Zaery baghlaninejad, 1385. Numeral model of sheet and turbulent seepage under concrete dams for analysis of lifting pressure and inside washing. MA thesis of hydraulic structures. Ahvaz Chamran university.
7. S.S.Rao, 1376. Finite elements method in engineering. Translated by Gh.Majzoubi and F.Fariba. First edition. Imam Hosein university publishment.
8. Abulfazl Shamsaie, 1378, hydraulic of water flow in porous environment, drainage engineering, Amirkabir university, first edition.
9. Seepage at dam's foundation and methods of controlling it- national committee of Iran's big dams- 1375, journal number 6.
10. G.Mahjoubi. study of seepage under an earth dam with finite elements method.
11. Principles of geotechnique engineering, Brajam, Das, translated by Shapour Tahouni.
12. GeoSlope Help Documents,(2004) "Seepage Modeling with SEEP/W", First Edition
13. samani, J-I.M.V., and J.M. V., Sarnani., and .M., Shaiannejad. (2003) "Reservoir Routing Using Steady and Unsteady Flow through Rock fill Dams",. Div .ASCE,
14. Chawla, A. S., (1972), "Boundary Effects on Stability of Tinctures", .J of Hydra. Div, ASCE
15. Herrera, N. M., and G. K., Felton. (1991), " Hydraulics of now through Rock fill dams using sediment - water", Trans. Of ASCE
16. Miguel, A. M. AND Jams, N. L.,( 1982). "Seepage and Groundwater:., Copyright E levier's scientific publishing company.