Journal of Academic and Applied Studies
Vol. 3(5) May 2013, pp. 28-37
Available online @ www.academians.org

ISSN1925-931X

Thermal Entanglement and Quantum State Teleportation

Mohsen Balvasi[1], Enayatollah Taghavi moghadam[1], Farshad Baharvand[2]

[1]*Department of Electrical Engineering, Mahshahr branch, Islamic Azad University, Mahshahr, Iran.*

[2]*Department of physics, shahid chamran university of Ahvaz,Iran.*

Abstract

This study investigates thermal entanglement in a four-qubit spin chain of Heisenberg model XX with the presence of the Dzyaloshinski-Moriya (DM) anisotropic asymmetric interaction, an external magnetic field, and another parameter. Then, the spin chain was used as a quantum channel for transferring an unknown quantum state. Finally, fidelity concept was used to check the similarity between input state and output state which, in fact, shows the degree of success in teleportation. In this way, the effect of different parameters on the fidelity was determined. It was concluded that quantum channel comprised of four-qubit spin chain which has thermal equilibrium is an appropriate candidate for quantum transfer of information.

Keywords: Thermal entanglement, quantum teleportation, density matrix, spin chain, DM interaction, fidelity.

I. Introduction

Quantum entanglement is a strange feature of quantum mechanics and plays a basic role in quantum information processing. Entangled state is a state which cannot be shown based on tensor product of individual states of comprising subsystems. Two systems are called entangled when experimenting on one system can reveal information about the other one. This nonlocal quantum characteristic will make us able to do some tasks (e. g. quantum teleportation, quantum cryptography and quantum dens coding) which are impossible to do without it. quantum teleportation is a way in which entanglement makes it possible to transfer unknown states whit the least classic bits to a far distances. Spin chains are practical ways of saving and transferring quantum information they are one of the quantum systems in which measurement and control of entanglement is important. They can be used as quantum communication channel and also can be used in quantum computers (M. A. N sen and I. L. Chuang, 2000; G. Beneti *et. al*, 2004).

Journal of Academic and Applied Studies
Vol. 3(5) May 2013, pp. 28-37
Available online @ www.academians.org

ISSN1925-931X

According to Boltsman principle, a system with Hamiltonian of H, and thermal equilibrium with probability of $\frac{1}{Z}\, exp\left(\frac{-E_i}{k_B T}\right)$ is state of $|\varphi_i\rangle$, which is eigenvector corresponding to eigenvalue E_i. Density matrix of such systems is determined by

$$\rho(T) = \frac{1}{Z} exp\left(\frac{-H}{k_B T}\right) = \frac{1}{Z}\sum_{i=1}^{N} exp\left(\frac{-E_i}{k_B T}\right)\, |\varphi_i\rangle\langle\varphi_i|$$

Where T is the temperature, k_B Boltsman constant and Z partition function of the system. Entanglement obtained from this density matrix is called thermal entanglement. Recently, quantum teleportation using thermal entanglement state has become the focus of a lot of studies. For example, studies on Teleportation and thermal entanglement in two qubit Heisenberg XYZ spin chain with the DM interaction and the inhomogeneous magnetic field (D. Gao *et. al.*, 2010), Effects of DM interaction on thermal entanglement and teleportation via a two qubit Heisenberg XXZ spin chain under external magnetic field (J.T. Cai *et. al.*, 2010), Thermal entanglement and teleportation in a three qubit Heisenberg XXZ model with DM anisotropic asymmetric interaction (X. Li-Jun *et. al*, 2009), Thermal entanglement in a four qubit Heisenberg spin model with external magnetic fields (D. Ke *et. al.*, 2007), Thermal entanglement and teleportation of a thermally mixed entangled state of a Heisenberg chain through a Werner state (H. Li-Yuan and F. Mao-Fa, 2008), Quantum teleportation through a two-qubit Heisenberg XXZ Chain (G. Jin-Liang, 2008), Entanglement and teleportation through a two-qubit Heisenberg XXZ model with the Dzyaloshinski-Moriya interaction (J.L. Guo and H.S. Song, 2010) were done.

In this research investigates thermal entanglement in a four-qubit spin chain of Heisenberg model XX with the presence of DM interaction, an external magnetic field along the z-axes. Then, the spin chain was used as a quantum channel for transferring an unknown quantum state.

II. The Heisenberg model XX with DM interaction under external magnetic field

The model investigated in this study is a Heisenberg XX four-qubit spin chain with Z component DM interaction, and external magnetic field along the z-axes. The Hamiltonian for this model is

$$\hat{H} = \frac{J}{4}(\,\sigma_1^X\sigma_2^X + \sigma_1^Y\sigma_2^Y + \sigma_2^X\sigma_3^X + \sigma_2^Y\sigma_3^Y + \sigma_3^X\sigma_4^X + \sigma_3^Y\sigma_4^Y) + \frac{B}{2}\sigma_1^Z + \frac{B}{2}\sigma_2^Z + \frac{B}{2}\sigma_3^Z + \frac{B}{2}\sigma_4^Z + \frac{D}{4}\left(\sigma_1^X\sigma_2^y - \sigma_1^y\sigma_2^x + \sigma_2^X\sigma_3^y - \sigma_2^y\sigma_3^x + \sigma_3^X\sigma_4^y - \sigma_3^y\sigma_4^x\right) \qquad (1)$$

Where J, D are spin-spin and spin- orbit interactions. Eigenvalues for the above Hamiltonian

$$E_1 = E_2 = 0\ , \qquad E_3 = -2B\ , \qquad E_4 = 2B$$

Journal of Academic and Applied Studies
Vol. 3(5) May 2013, pp. 28-37
Available online @ www.academians.org

ISSN1925-931X

$$E_5 = -\frac{A}{2} \quad , \quad E_6 = \frac{A}{2} \quad , \quad E_7 = -\frac{A}{2}\sqrt{5}$$
$$E_8 = \frac{A}{2}\sqrt{5} \ , \ E_9 = -B - \frac{L}{4} \ , \ E_{10} = B - \frac{L}{4}$$
$$E_{11} = -B + \frac{L}{4} \ , \ E_{12} = B + \frac{L}{4} \ , \ E_{13} = -B - \frac{P}{4}$$
$$E_{14} = B - \frac{P}{4} \ , \ E_{15} = -B + \frac{P}{4} \ , \ E_{16} = B + \frac{P}{4}$$

Corresponding eigenvectors for the above eigenvalues

$$\Psi_1 = R(\frac{Q^2}{K^2}|0001\rangle + \frac{Q}{K}|1010\rangle + |1011\rangle)$$
$$\Psi_2 = \frac{1}{\sqrt{2}}(-|1010\rangle + |0101\rangle), \qquad \Psi_3 = |1111\rangle \ , \ \Psi_4 = |0000\rangle$$
$$\Psi_5 = S(-\frac{Q^2}{K^2}|0001\rangle - \frac{AV}{K^2}|1100\rangle - \frac{V}{A}|1010\rangle + |1011\rangle)$$
$$\Psi_6 = W(-\frac{Q^2}{K^2}|0001\rangle + \frac{AV}{K^2}|1100\rangle + \frac{V}{A}|1011\rangle + |1011\rangle)$$
$$\Psi_7 = N(\frac{Q^2}{K^2}|0001\rangle + \frac{\sqrt{5}AV}{K^2}|1100\rangle + \left(2 - \frac{4D}{K}\right)|1011\rangle + \left(2 - \frac{4D}{K}\right)|1010\rangle - \frac{\sqrt{5}V}{A}|0101\rangle + |1011\rangle)$$
$$\Psi_8 = M(\frac{Q^2}{K^2}|0001\rangle - \frac{\sqrt{5}AV}{K^2}|1100\rangle + \left(2 - \frac{4D}{K}\right)|1011\rangle + \left(2 - \frac{4D}{K}\right)|1010\rangle + \frac{\sqrt{5}V}{A}|0101\rangle + |1011\rangle)$$
$$\Psi_9 = C''(-\frac{iC(L)}{4K^3}|1001\rangle - \frac{F}{2K^2}|0111\rangle - \frac{i(L)}{2K}|1101\rangle + |1110\rangle)$$
$$\Psi_{10} = C''(-\frac{iC(L)}{4K^3}|0100\rangle - \frac{F}{2K^2}|0010\rangle - \frac{i(L)}{2K}|1000\rangle + |0110\rangle)$$
$$\Psi_{11} = C'(\frac{iC(L)}{4K^3}|1001\rangle - \frac{F}{2K^2}|0111\rangle + \frac{i(L)}{2K}|1101\rangle + |1110\rangle)$$
$$\Psi_{12} = C'(\frac{iC(L)}{4K^3}|0100\rangle - \frac{F}{2K^2}|0010\rangle + \frac{i(L)}{2K}|1000\rangle + |0110\rangle)$$
$$\Psi_{13} = G(-\frac{iF(P)}{4K^3}|1001\rangle - \frac{C}{2K^2}|0111\rangle - \frac{i(P)}{2K}|1101\rangle + |1110\rangle)$$

Journal of Academic and Applied Studies
Vol. 3(5) May 2013, pp. 28-37
Available online @ www.academians.org

ISSN1925-931X

$$\Psi_{14} = G(-\frac{iF(P)}{4K^3}|0100\rangle - \frac{C}{2K^2}|0010\rangle - \frac{i(P)}{2K}|1000\rangle + |0110\rangle)$$
$$\Psi_{15} = Y(\frac{iF(P)}{4K^3}|1001\rangle - \frac{C}{2K^2}|0111\rangle + \frac{i(P)}{2K}|1101\rangle + |1110\rangle)$$
$$\Psi_{16} = Y(\frac{iF(P)}{4K^3}|0100\rangle - \frac{C}{2K^2}|0010\rangle + \frac{i(P)}{2K}|1000\rangle + |0110\rangle)$$

Where $R, S, W, N, M, C', C'', G, Y$ are its normalization coefficients. The other parameters in the above equations are

$$A = \sqrt{D^2 + J^2} \quad , \quad V = i\mathrm{D} + J \quad , \quad \mathrm{Q} = \mathrm{D} - iJ \quad , \quad K = \mathrm{D} + iJ$$

$$P = \sqrt{6D^2 + 6J^2 + 2\sqrt{5}\sqrt{(D^2 + J^2)^2}} \quad , \quad L = \sqrt{6D^2 + 6J^2 - 2\sqrt{5}\sqrt{(D^2 + J^2)^2}}$$
$$C = D^2 + J^2 + \sqrt{5}\sqrt{(D^2 + J^2)^2} \quad , \quad F = D^2 + J^2 - \sqrt{5}\sqrt{(D^2 + J^2)^2}$$

By considering eigenvalues and eigenvectors, the density matrix of the system is specified. Reduced density matrix for the second and the third portion by computing partial trace for these two portions is

$$\rho(T)_{14} = \frac{1}{\mathrm{Z}}\begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & \xi & \eta & 0 \\ 0 & \eta^* & \upsilon & 0 \\ 0 & 0 & 0 & \gamma \end{pmatrix} \qquad (2)$$

With

$$\mu = \frac{1}{5\sqrt{5}\mathrm{g}} e^{-\frac{B}{T}}(-5\mathrm{g}(Cosh[\frac{L}{4T}] - Cosh[\frac{P}{4T}]) + \sqrt{5}\mathrm{g}(2Cosh[\frac{B}{T}](4 + Cosh[\frac{\sqrt{5}A}{2T}]) + 5(Cosh[\frac{L}{4T}] + Cosh[\frac{P}{4T}]) + 4Sinh[\frac{B}{T}]Sinh[\frac{\sqrt{5}A}{4T}]^2))$$

$$\xi = \frac{1}{5\sqrt{5}\mathrm{g}}(2\sqrt{5}\mathrm{g} + 5\sqrt{5}\mathrm{g}Cosh[\frac{A}{2T}] + 3\sqrt{5}\mathrm{g}Cosh[\frac{\sqrt{5}A}{2T}] + 5Cosh[\frac{B}{T}](C\ Cosh[\frac{L}{4T}] + (-\mathrm{g} + \sqrt{5}\mathrm{g})Cosh[\frac{P}{4T}]))$$

$$\eta = (\mathrm{V}(-\frac{5\mathrm{ALP}}{2}\mathrm{Sinh}[\frac{\mathrm{A}}{2T}] + \sqrt{5}(\frac{\mathrm{ALP}}{2}\mathrm{Sinh}[\frac{\sqrt{5}\mathrm{A}}{2T}] + 2\sqrt{2}\mathrm{g}\mathrm{Cosh}[\frac{B}{T}](-\frac{\mathrm{P}}{\sqrt{2}}\mathrm{Sinh}[\frac{\mathrm{L}}{4T}] + \frac{\mathrm{L}}{\sqrt{2}}\mathrm{Sinh}[\frac{\mathrm{P}}{4T}]))))/(5\mathrm{K}^2\sqrt{2}\,\mathrm{L}\sqrt{2}\,\mathrm{P})$$

$$\upsilon = \frac{1}{5\sqrt{5}\mathrm{g}}(2\sqrt{5}\mathrm{g} + 5\sqrt{5}\mathrm{g}Cosh[\frac{A}{2T}] + 3\sqrt{5}\mathrm{g}Cosh[\frac{\sqrt{5}A}{2T}] + 5Cosh[\frac{B}{T}](C\ Cosh[\frac{L}{4T}] + (-\mathrm{g} + \sqrt{5}\mathrm{g})Cosh[\frac{P}{4T}]))$$

Journal of Academic and Applied Studies
Vol. 3(5) May 2013, pp. 28-37
Available online @ www.academians.org

ISSN1925-931X

$$\gamma = \{(3 + 5e^{\frac{2B}{T}})A^2 + 2A^2 Cosh[\frac{\sqrt{5}A}{2T}]$$
$$+e^{\frac{B}{T}}((5g - \sqrt{5}g)Cosh[\frac{L}{4T}] + (5g + \sqrt{5}g)Cosh[\frac{P}{4T}])\}/5A^2$$

Partition function system is

$$Z = 2(1 + Cosh[\frac{2B}{T}] + Cosh[\frac{A}{2T}] + Cosh[\frac{\sqrt{5}A}{2T}] + 2Cosh[\frac{B}{T}](Cosh[\frac{L}{4T}] + Cosh[\frac{P}{4T}])) \qquad (3)$$

To quantify the amount of entanglement of a two- portion system with density matrix ρ, we consider the concurrence which is defined as (W. K. Wootters, 2001)

$$C(\hat{\rho}_{AB}) = max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

Where λ_i are the square roots of the eigenvalues of the matrix $R = \rho_{AB}(\sigma_y^A \otimes \sigma_y^B)\rho^*_{AB}(\sigma_y^A \otimes \sigma_y^B)$ in descending order. σ_y is component y of Pauli matrix and ρ^*_{AB} is complex conjugation in the computational basis $|0\ 0\rangle, |0\ 1\rangle, |1\ 0\rangle, |1\ 1\rangle$. The concurrence range from zero to one, zero value is a disentangled state and one value for a maximally entangled state.
The concurrence for each density matrix in form of ρ_{14} is

$$C(\hat{\rho}) = \frac{2}{Z} max\{0, |\eta| - \sqrt{\mu\, \gamma}\} \qquad (4)$$

System thermal entanglement can be analyzed qualitatively with regard to preceding concurrence function.
In fig.1 concurrence versus temperature for different magnetic fields is plotted. The figure show that by increasing the external magnetic field, the concurrence and critical temperature rise. By increasing the temperature, the amount of concurrence decreases. In fig-2 demonstrate concurrence versus temperature for different DM interaction. It shows that by increasing parameter DM, concurrence and critical temperature increase. Any increase in the temperature causes concurrence to fall down, except when the system is separable at zero temperature.

Journal of Academic and Applied Studies
Vol. 3(5) May 2013, pp. 28-37
Available online @ www.academians.org

ISSN1925-931X

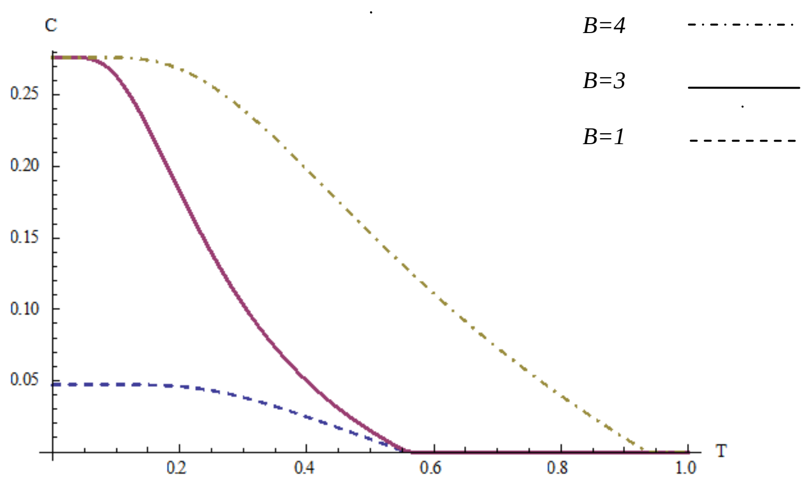

Fig.1.The concurrence versus temperature for different magnetic fields and J=D=5

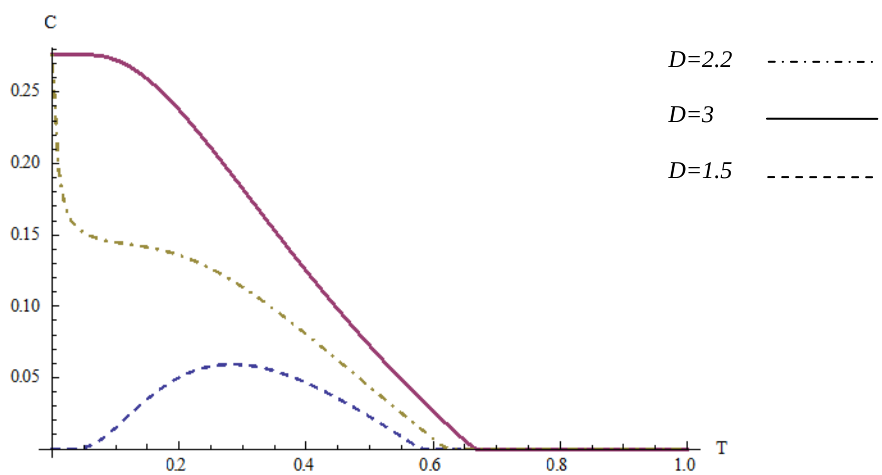

Fig.2.The concurrence versus temperature for different DM interaction and J=B=3

III. Quantum teleportation and fidelity

Now we want to use the Heisenberg four- qubit spin chain discussed about as a quantum channel for transferring a state to far distances. Suppose that unknown input state for transfer is a two-qubit arbitrary pure state like this

Journal of Academic and Applied Studies
Vol. 3(5) May 2013, pp. 28-37
Available online @ www.academians.org

ISSN1925-931X

$$|\varphi_{in}\rangle = \cos\frac{\theta}{2}|1\,0\rangle + e^{i\phi}\sin\frac{\theta}{2}|\,01\rangle \qquad (5)$$

Where angles $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$ are the polar and directional angles respectively. By using Quantum teleportation protocol, the output state with density matrix is determined (G. Bowen and S. Bose, 2001)

$$\rho_{out} = \sum_{i,j=0,x,y,z} p_{ij}\,(\sigma_i \otimes \sigma_j)\rho_{in}(\sigma_i \otimes \sigma_j) \qquad (6)$$

Where σ_0 unitary matrix and $\sigma_i(i = x, y, z)$ Pauli matrix. ρ_{in} density matrix for input state and $p_{ij} = Tr[E^i \rho(T)].Tr[E^j \rho(T)].\ (\sum_{ij} p_{ij} = 1)$
E^0, E^x, E^y, E^z matrixes, based on for bell states, are

$$E^0 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad E^x = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

$$E^z = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad E^y = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

After straightforward calculation, the output density matrix is

$$\rho(T)_{out} = \begin{pmatrix} \mu' & 0 & 0 & 0 \\ 0 & \xi' & \eta' & 0 \\ 0 & \eta'^{*} & \upsilon' & 0 \\ 0 & 0 & 0 & \mu' \end{pmatrix} \qquad (7)$$

To determine the quality of transfer state, fidelity is defined as

$$F(\rho_{in}, \rho_{out}) = \left[Tr\left(\sqrt{\sqrt{\rho_{in}}\,(\rho_{out})\,\sqrt{\rho_{in}}}\right)\right]^2 \qquad (8)$$

The values for fidelity range from zero to one. When $F(\rho_{in}, \rho_{out}) = 0$ all the information transfer is destroyed. When $F(\rho_{in}, \rho_{out}) = 1$ information transfer is completely successful and the output state is completely similar to the input state. Fidelity is the result of one of the measurements based on Bell basis that the receiver of the message determines. But average fidelity shows the average of all measurements that are done by the receivers.

Journal of Academic and Applied Studies
Vol. 3(5) May 2013, pp. 28-37
Available online @ www.academians.org

ISSN1925-931X

In this study, we only investigate the fidelity concept. Having input and output density matrix and using the above equation (8), we determine the fidelity (refer to appendix). In fig.3 demonstrate fidelity versus temperature for different magnetic fields. It shows that the larger the magnetic fields the more the fidelity between input and output state and subsequently the less distorted information. In fig.4 fidelity versus parameter DM for different temperature is plotted. It shows that increasing the temperature makes the fidelity weaker. By increasing DM, the fidelity, at first, decreases then it became a constant value and subsequently it increases.

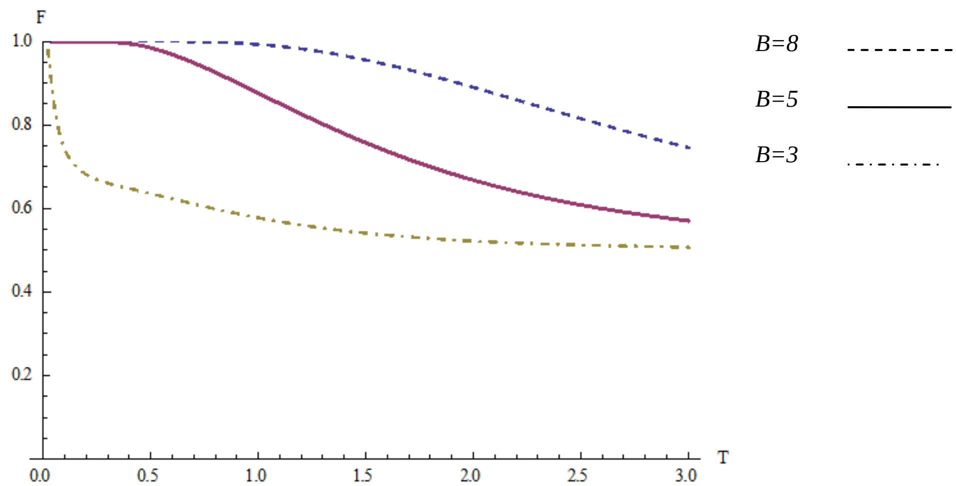

Fig.3.The fidelity versus temperature for different magnetic fields, D=2, J=3 and $\theta = \frac{\pi}{2}$.

Journal of Academic and Applied Studies
Vol. 3(5) May 2013, pp. 28-37
Available online @ www.academians.org

ISSN1925-931X

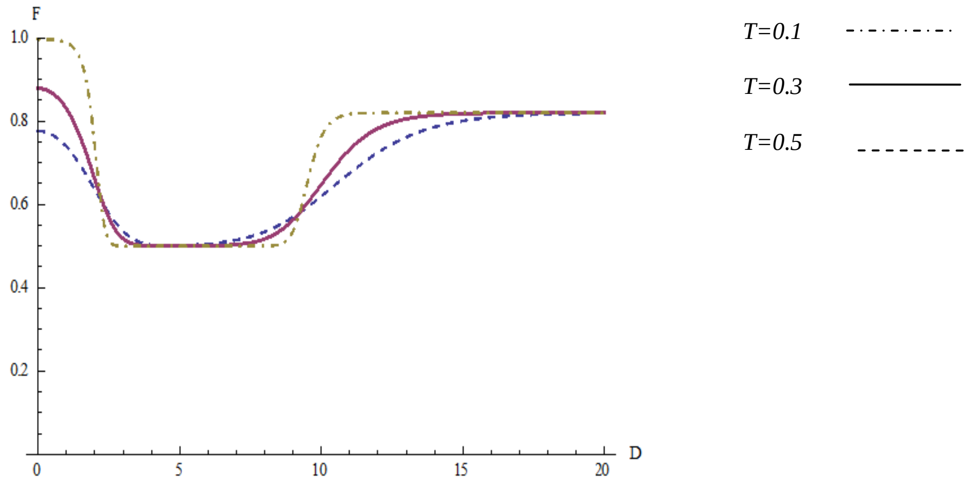

Fig.4. The fidelity versus parameter DM for different temperature, J=B=3 and $\theta = \frac{\pi}{2}$.

.

IV. Conclusions

We conclude that quantum channel comprised of four-qubit spin chain which has thermal equilibrium is an appropriate candidate for quantum information transfer. when external magnetic field and spin-orbit interaction are enhanced, it cause a better transfer in the channel.

References

M. A. N sen and I. L. Chuang, “Quantum computation and quantum information”, Cambridge University Press 116 (2000).

G. Beneti, G. Casati, G. Strini, “principles of quantum computation and information”, universita degli studi dell insubria, Italy (2004).

D. Gao, Sh. Zhaog, D. Zhu, F. Wang Hong, “Teleportation and thermal entanglement in two qubit heisenberg XYZ spin chain with the DM interaction and the inhomogeneous magnetic field”, Chin. Phys. B. **19**, 090313, 1-5 (2010).

J.T. Cai, A. Abliz, G. F. Zang, Y.K. Bai, “Effects of DM interaction on thermal entanglement and teleportation via a two qubit heisenberg XXZ spin chain under external magnetic field”,Optcom, **283**, 4415- 4421 (2010).

X. Li-Jun, Z. Deng-Yu, T. Shi-Qing, Z. Xiao-Gui, G. Feng "Thermal entanglement and teleportation in a three qubit heisenberg XXZ model with DM anisotropic antisymmetric interaction", Chin. Phys. B. **18**, 3203-3209 (2009).

D. Ke, Z. Bin, Q. Wan, "Thermal entanglement in a four qubit heisenberg spin model with external magnrtic fields", Phys. Lett. A, **362**, 381-389 (2007).

H. Li-Yuan and F. Mao-Fa "Thermal entanglement and teleportation of a thermally mixed entangled state of a Heisenberg chain through a Werner state", Chin. Phys. B. 17, 2339-2345 (2008).

G. Jin-Liang, N. Jing, and S. He-Shan, "Quantum teleportation through a two-qubit Heisenberg XXZ Chain", Commun. Theor. Phys. **49**, 1435–1438 (2008).

J.L. Guo and H.S. Song, " Entanglement and teleportation through a two-qubit Heisenberg XXZ model with the Dzyaloshinskii-Moriya interaction", Eur. Phys. J. D. **56**, 265–269 (2010).

W. K. Wootters, "Entanglement of formation and concurrence", Quantum. Inf.

Comput. **1**, 27- 44 (2001).

G. Bowen and S. Bose, "Teleportation as a Depolarizing Quantum Channel, Relative Entropy and Classical Capacity", Phys. Rev. Lett. **87**, 267901 (2001).